Full Mueller AW-Projection

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URSI 2019

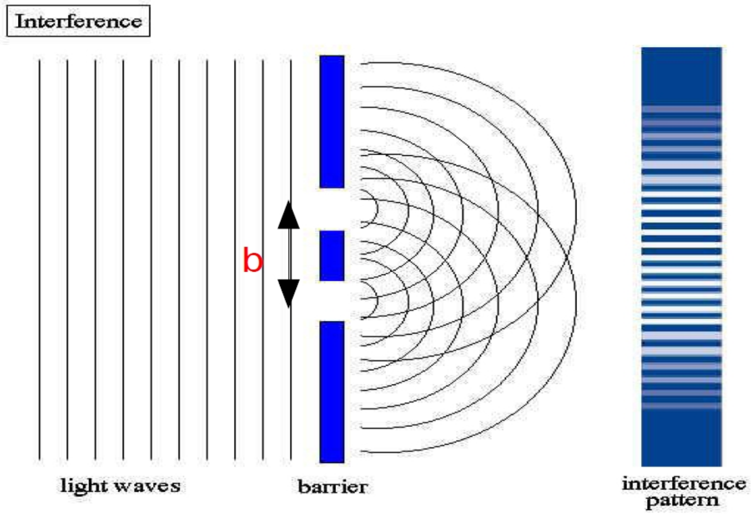
Outline

What is AW-projection ?

- Interferometric imaging recap
- Wide-field wide-band imaging in CASA
- Role of the Antenna Primary Beam in Imaging
- Full Mueller AW Projection algorithm

Interferometry - I

Young's double slit experiment



2D Fourier transform :

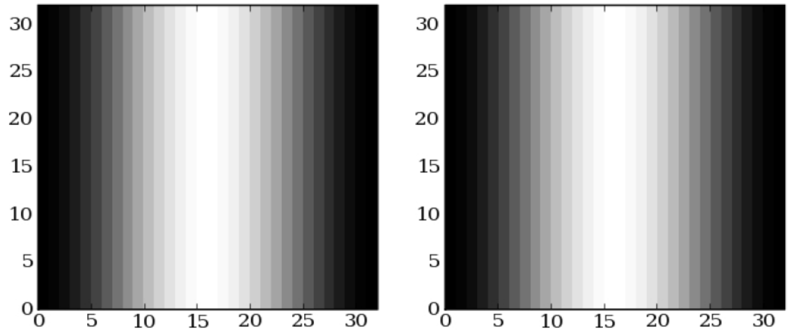
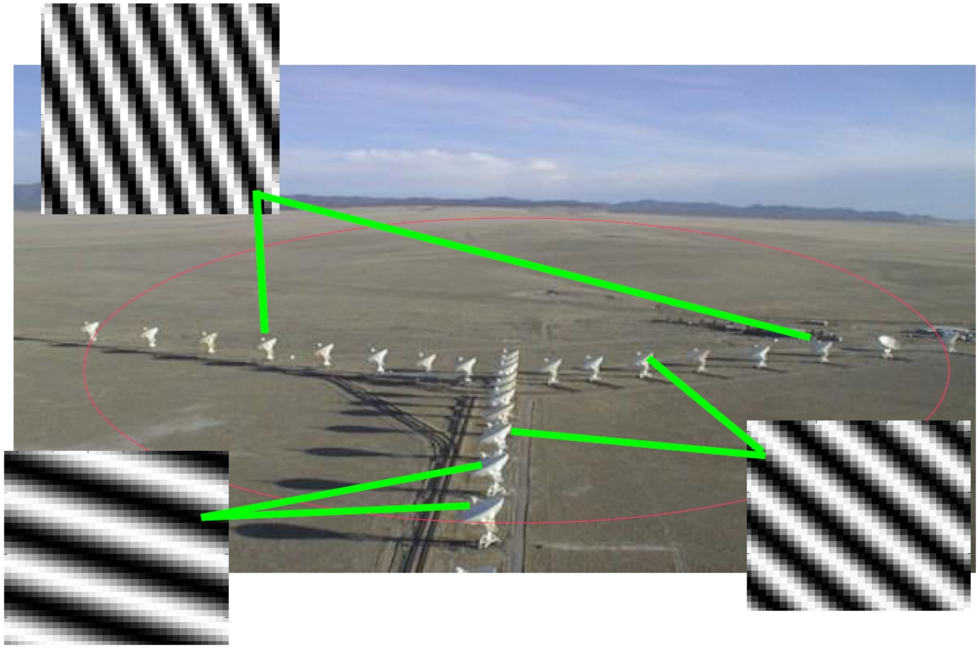


Image = sum of cosine 'fringes'.

Each antenna-pair measures the parameters of one 'fringe'.



Parameters of a Fringe :

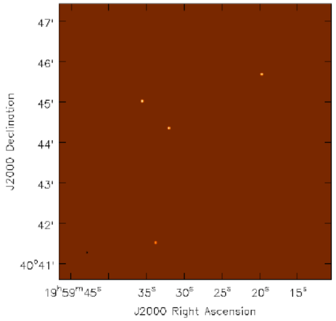
Amplitude, Phase

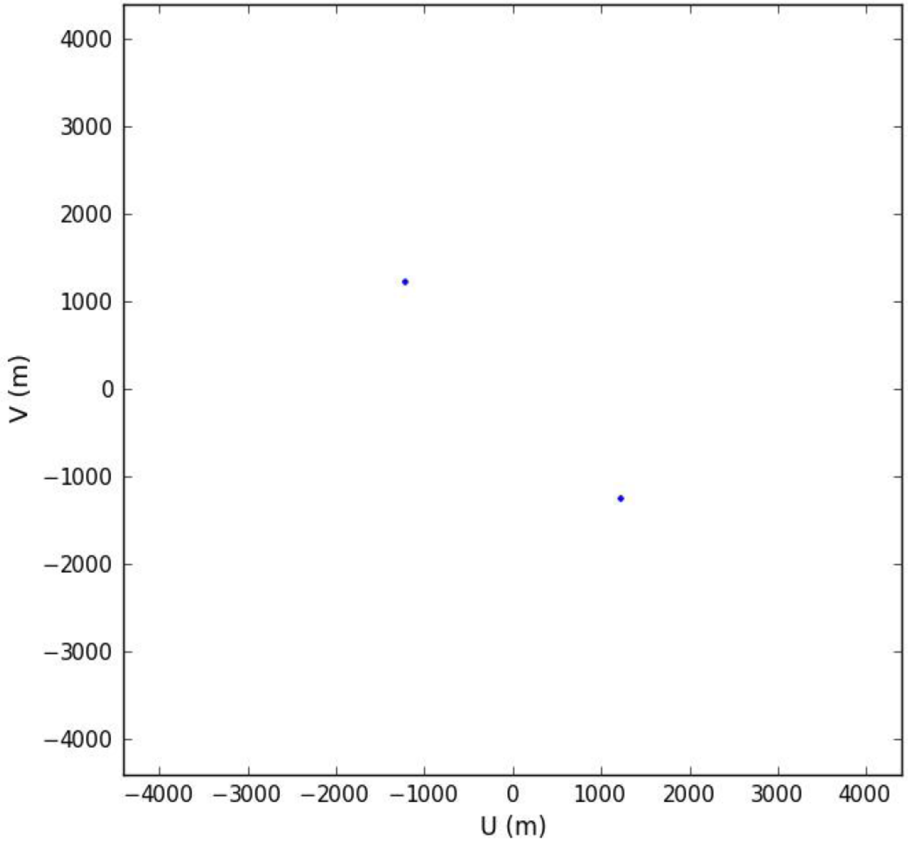
Orientation, Wavelength

Courtesy: U.Rau

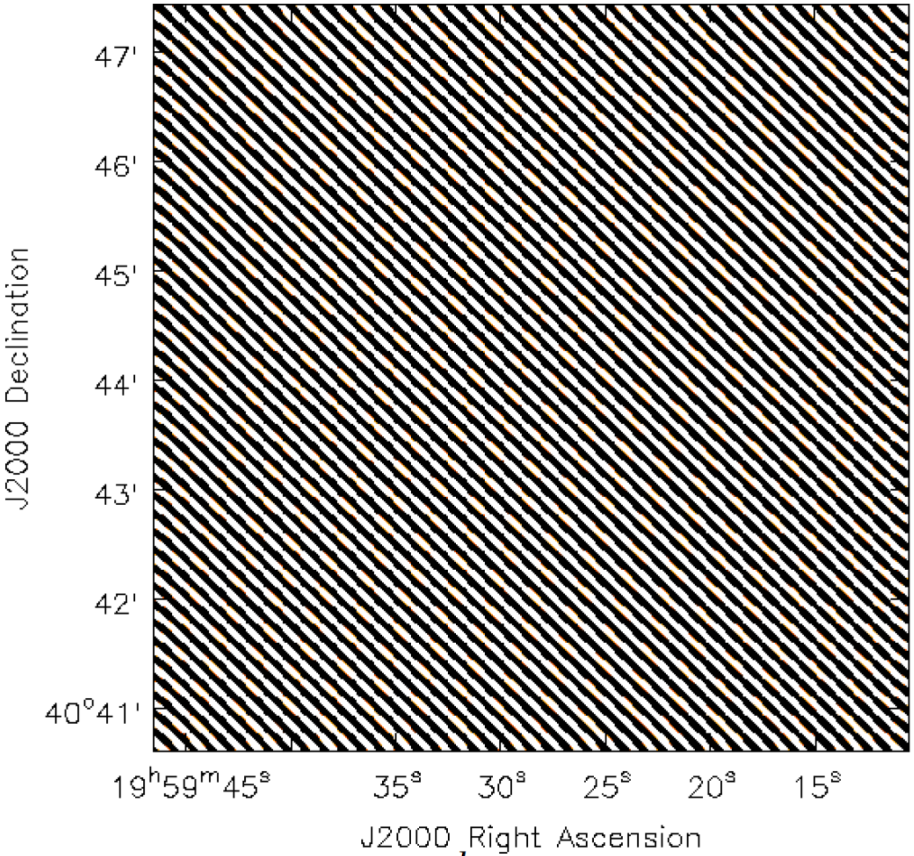
$$\begin{bmatrix} u \\ v \\ w \end{bmatrix} = \frac{1}{\lambda} \begin{bmatrix} R(h,\theta) \end{bmatrix} \begin{bmatrix} \delta x \\ \delta y \\ \delta z \end{bmatrix}$$

Image of the sky using 2 antennas





$S(u,v)$

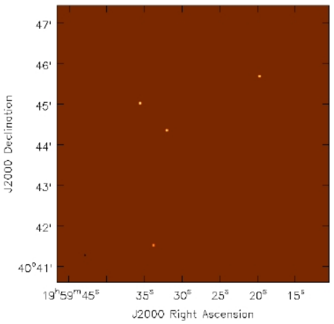


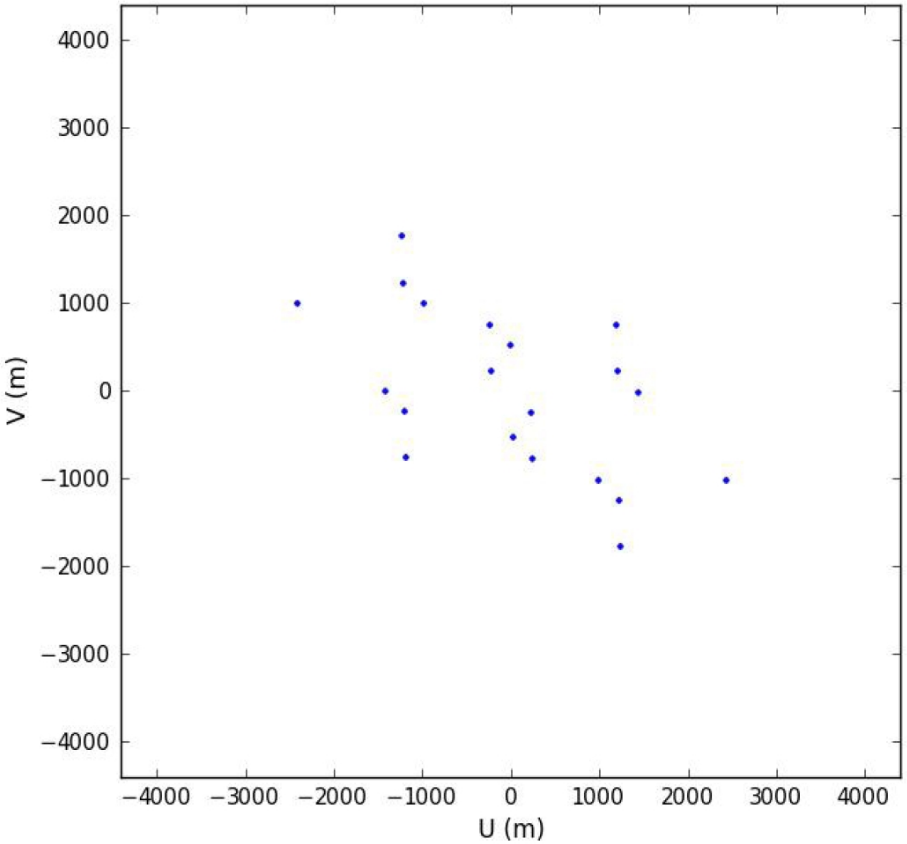
$I^{obs}(l,m)$

Courtesy: U.Rau

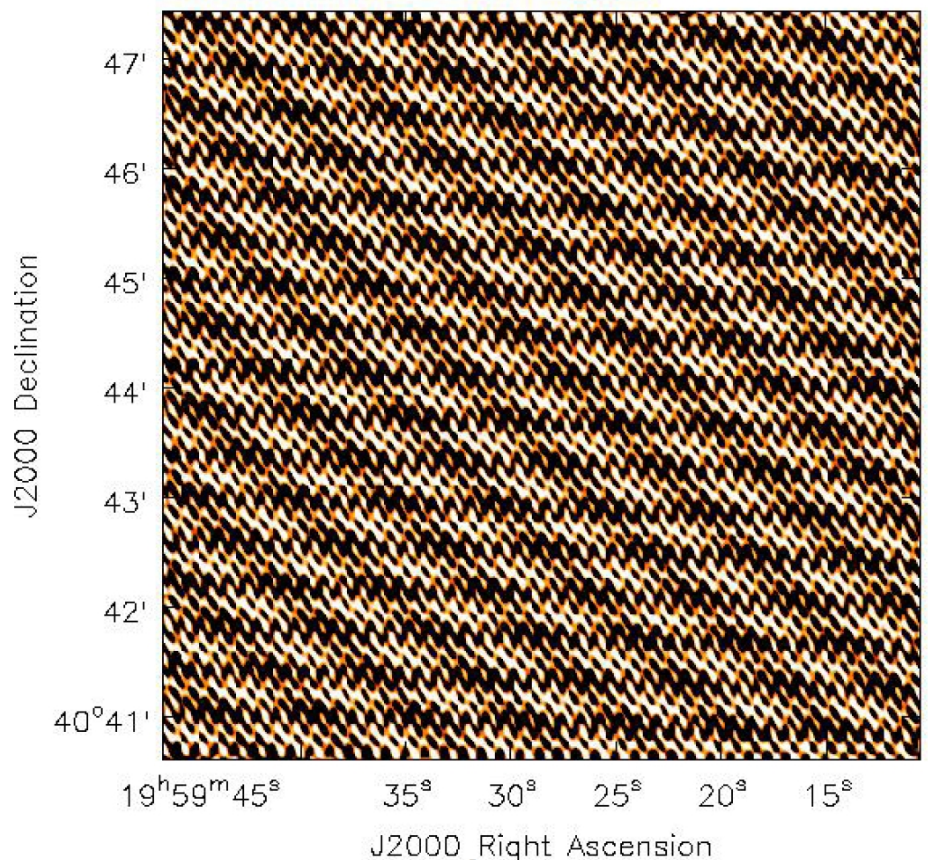
$$\begin{bmatrix} u \\ v \\ w \end{bmatrix} = \frac{1}{\lambda} \left[R(h,\theta) \right] \begin{bmatrix} \delta x \\ \delta y \\ \delta z \end{bmatrix}$$

Image of the sky using 5 antennas





$S(u,v)$

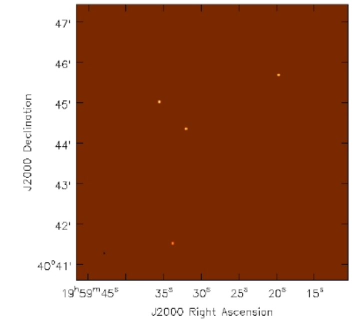


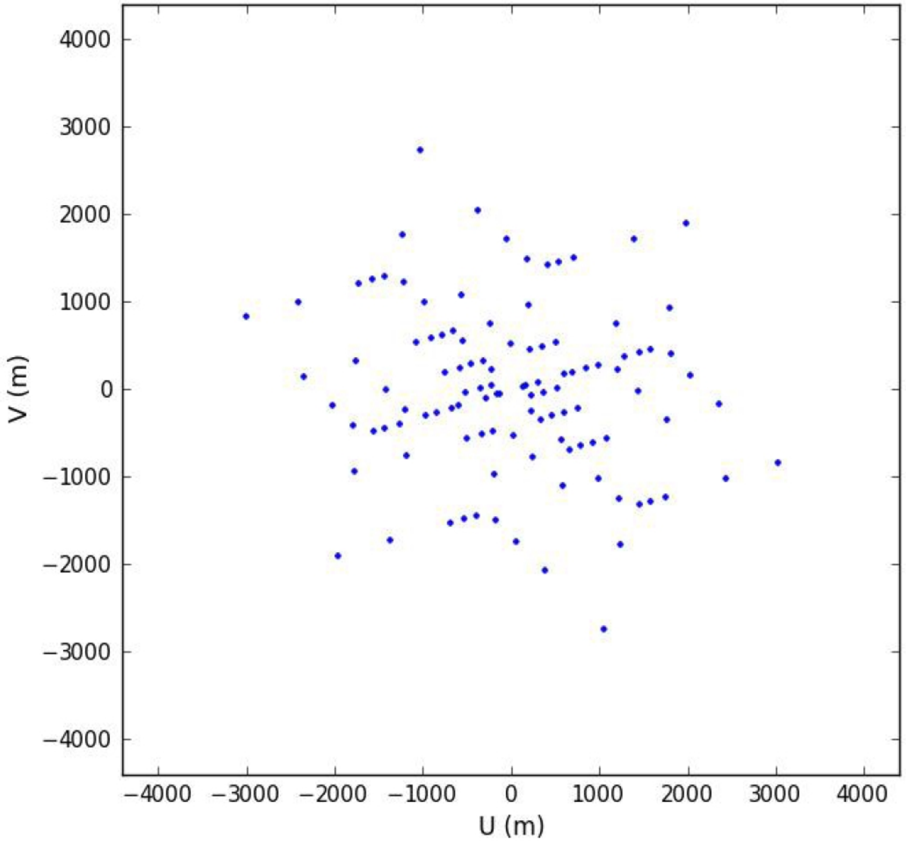
$I^{obs}(l,m)$

Courtesy: U.Rau

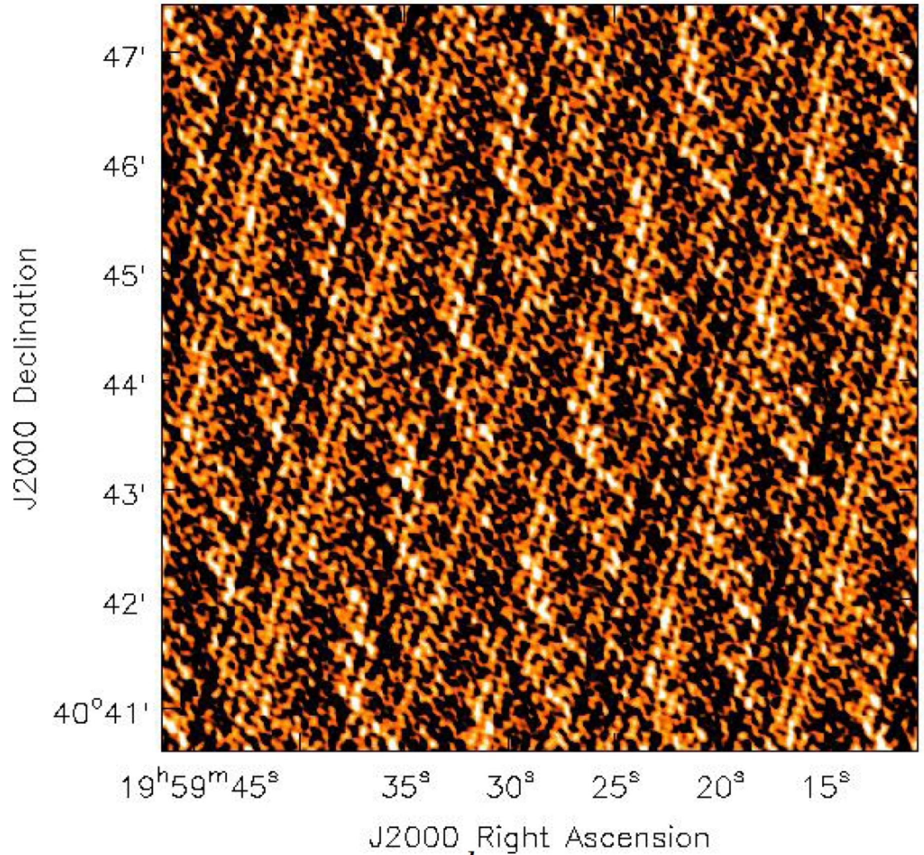
$$\begin{bmatrix} u \\ v \\ w \end{bmatrix} = \frac{1}{\lambda} \begin{bmatrix} R(h,\theta) \end{bmatrix} \begin{bmatrix} \delta x \\ \delta y \\ \delta z \end{bmatrix}$$

Image of the sky using 11 antennas





$S(u,v)$

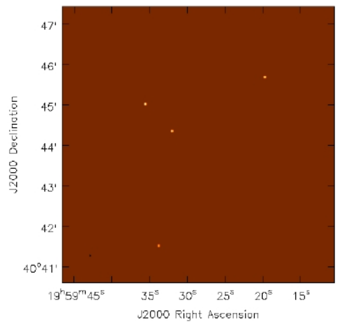


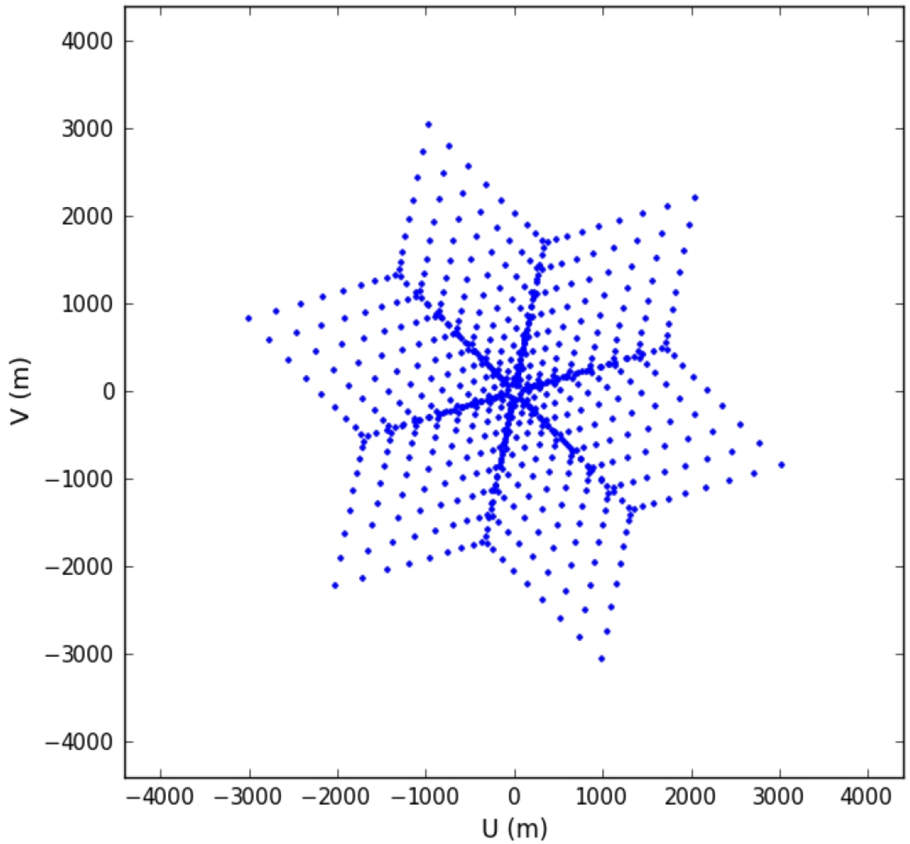
$I^{obs}(l,m)$

Courtesy: U.Rau

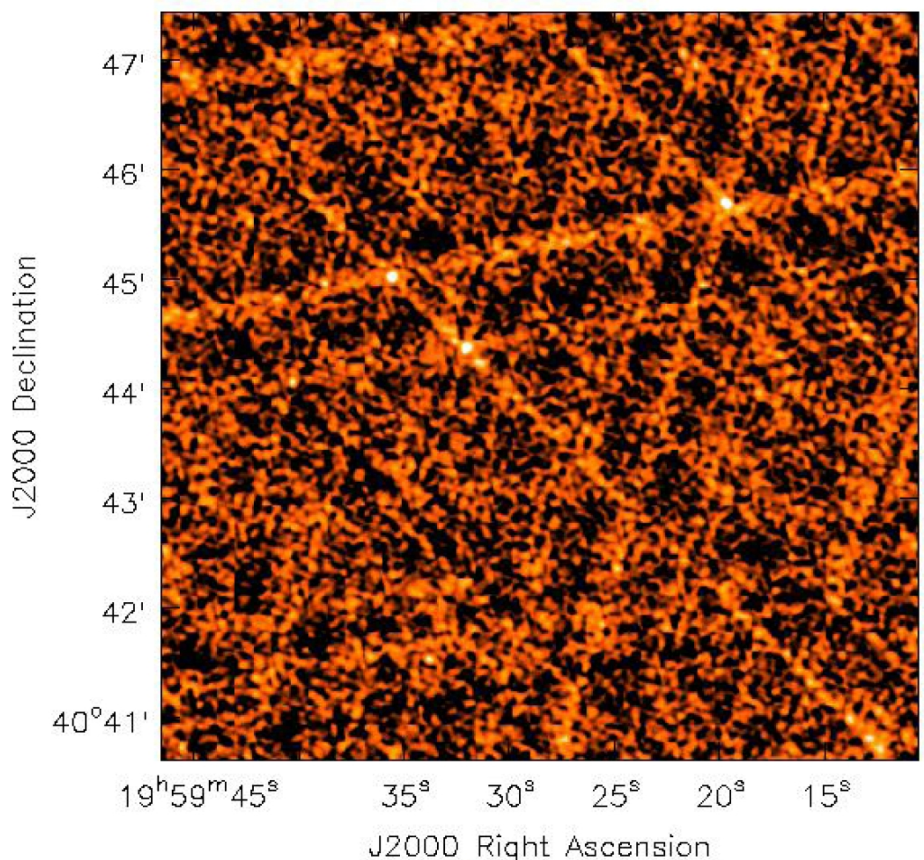
$$\begin{bmatrix} u \\ v \\ w \end{bmatrix} = \frac{1}{\lambda} \begin{bmatrix} R(h,\theta) \end{bmatrix} \begin{bmatrix} \delta x \\ \delta y \\ \delta z \end{bmatrix}$$

Image of the sky using 27 antennas





$S(u,v)$

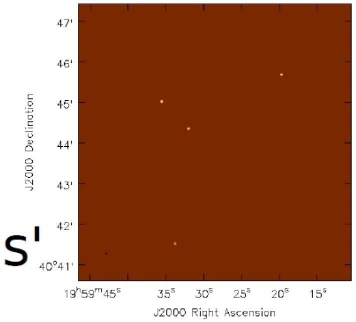


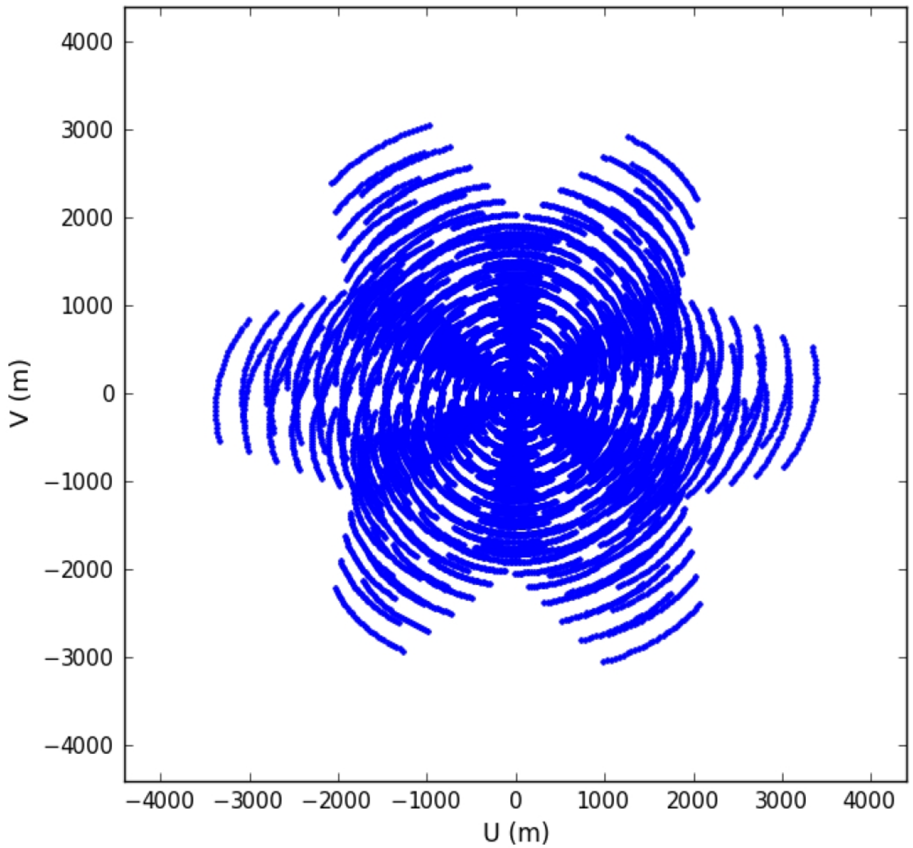
$I^{obs}(l,m)$

Courtesy: U.Rau

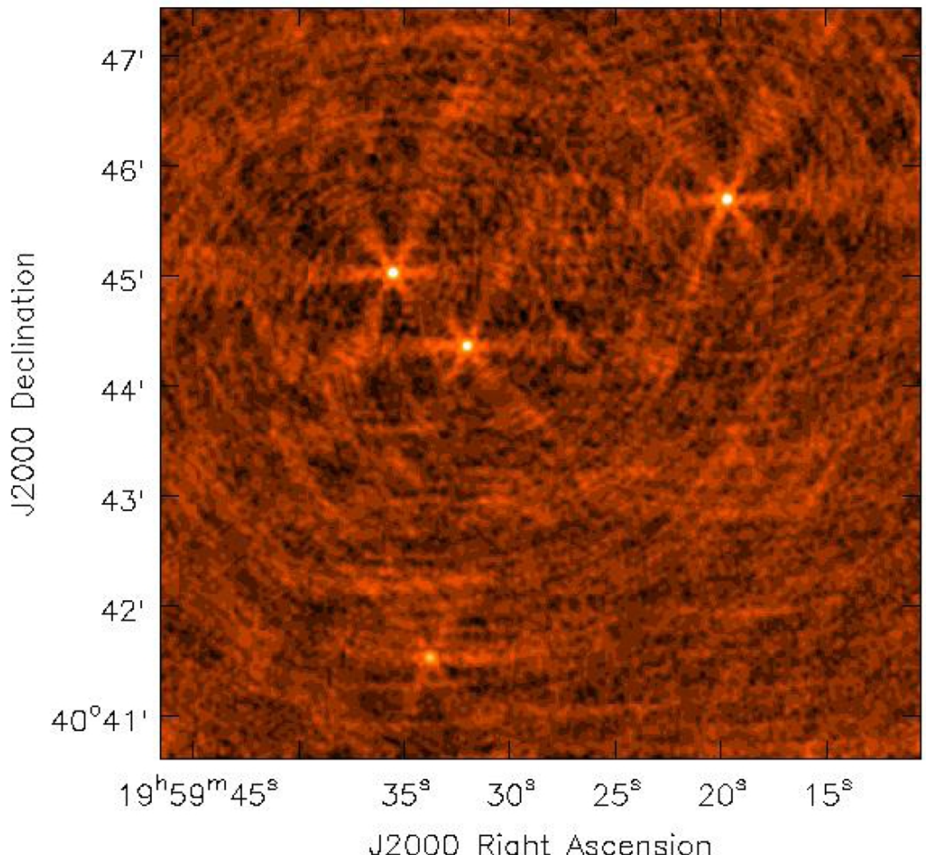
$$\begin{bmatrix} u \\ v \\ w \end{bmatrix} = \frac{1}{\lambda} \left[R(h,\theta) \right] \begin{bmatrix} \delta x \\ \delta y \\ \delta z \end{bmatrix}$$

Image of the sky
using 27 antennas
over 2 hours
'Earth Rotation Synthesis'





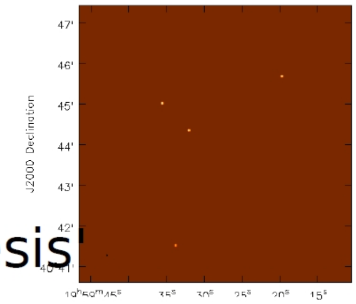
$S(u,v)$

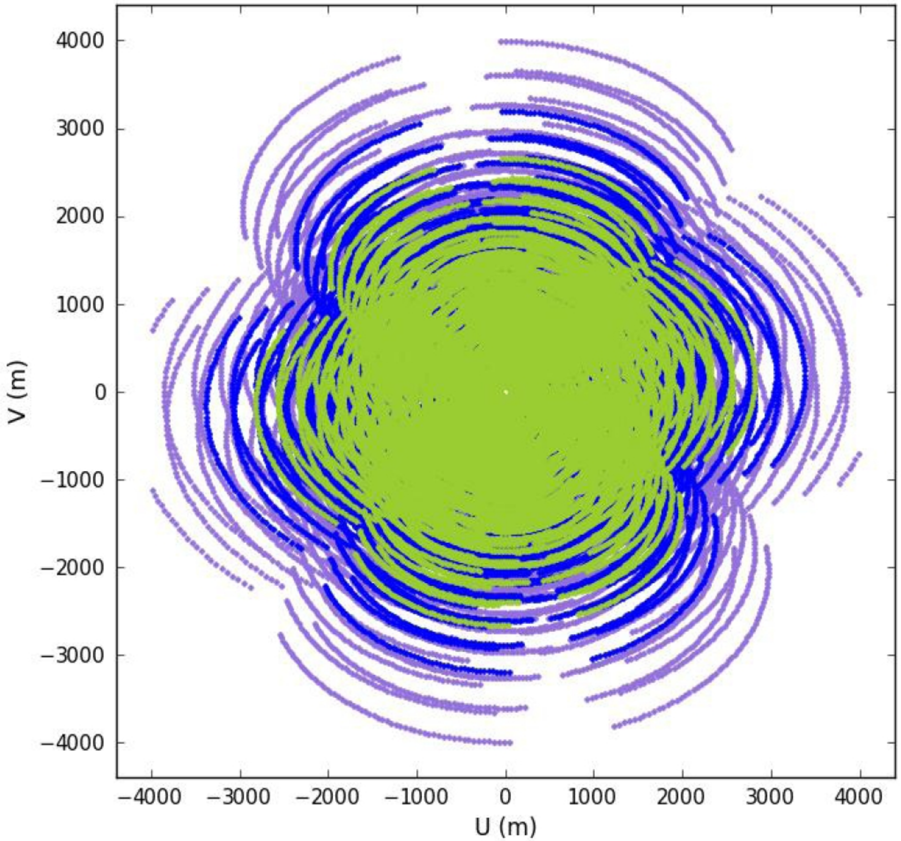


$I^{obs}(l,m)$ Courtesy: U.Rau

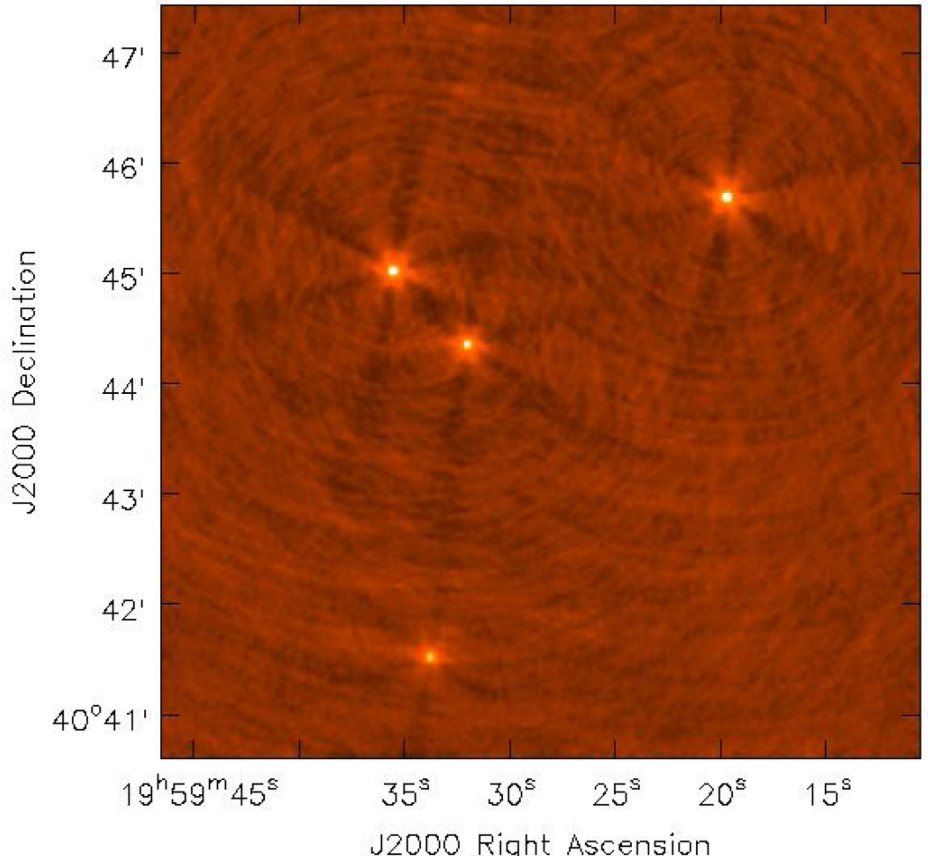
$$\begin{bmatrix} u \\ v \\ w \end{bmatrix} = \frac{1}{\lambda} \begin{bmatrix} R(h,\theta) \end{bmatrix} \begin{bmatrix} \delta x \\ \delta y \\ \delta z \end{bmatrix}$$

Image of the sky
using 27 antennas
over 4 hours, 3 freqs
'Multi-Frequency Synthesis'





$S(u,v)$

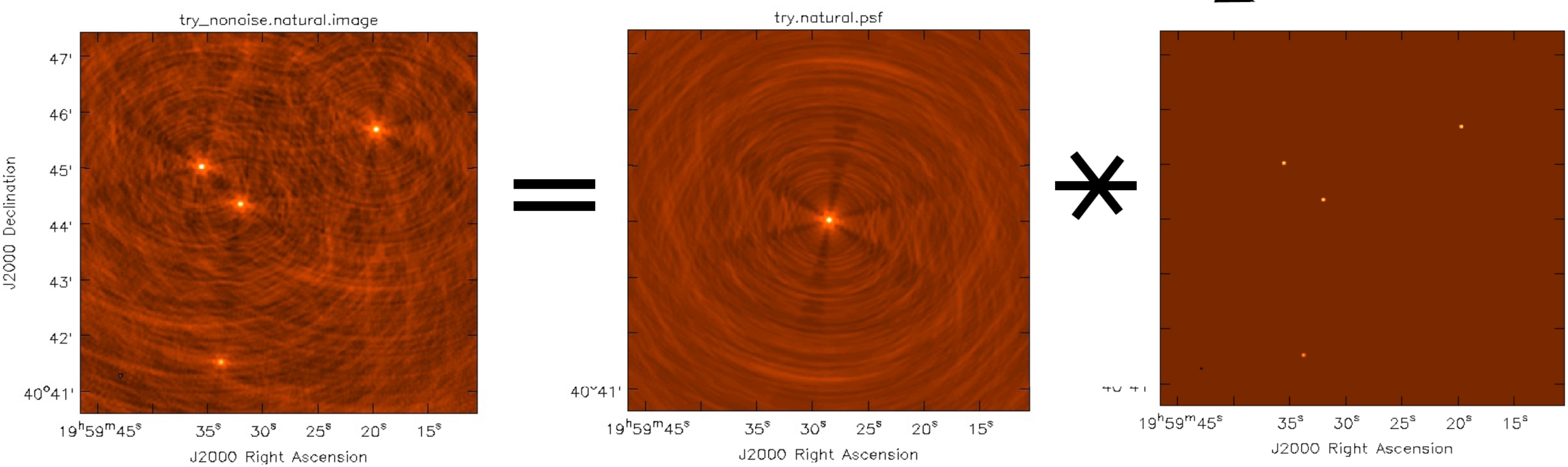


$I^{obs}(l,m)$

Courtesy: U.Rau

Interferometric Imaging

$$I^{obs}(l,m) = I^{PSF}(l,m) * I^{sky}(l,m)$$



Courtesy: U.Rau

Measurement Equation

$$\vec{V}_{ij}^{obs}(u,v) = G_{ij}^{DI} \int\int P_{ij}(s,\nu,t) I(s,\nu) e^{-2\pi i (u_{ij} l + v_{ij} m + w_{ij}(\sqrt{1-l^2-m^2}-1))}$$

Observed Visbilities
(Raw Data)

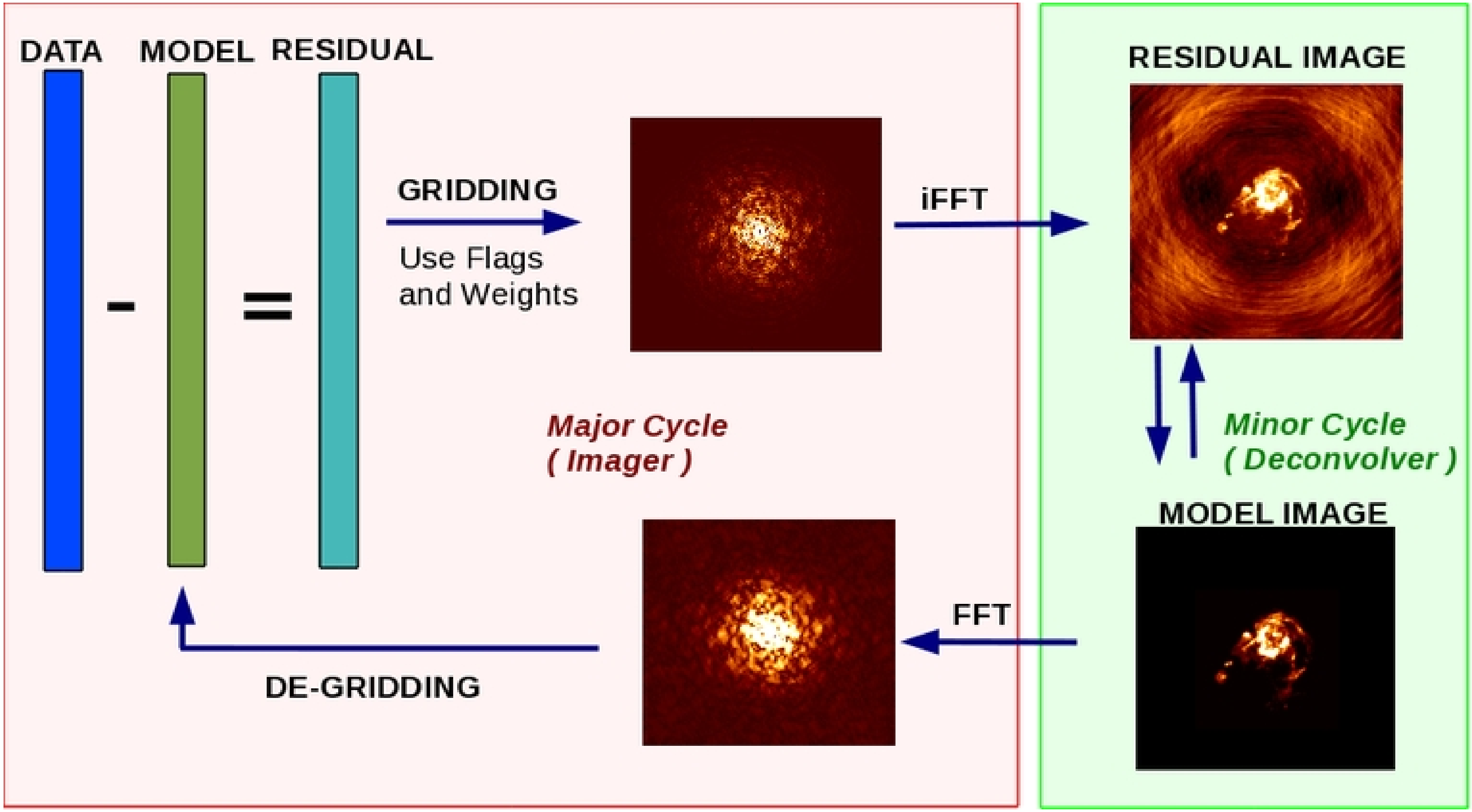
Calibration – Direction independent gains

Primary Beam of baseline – A Projection (gridding correction, major cycle)

Sky Brightness (deconvolution source modeling MT-MFS)

W-Projection (gridding correction, major cycle)

Imaging

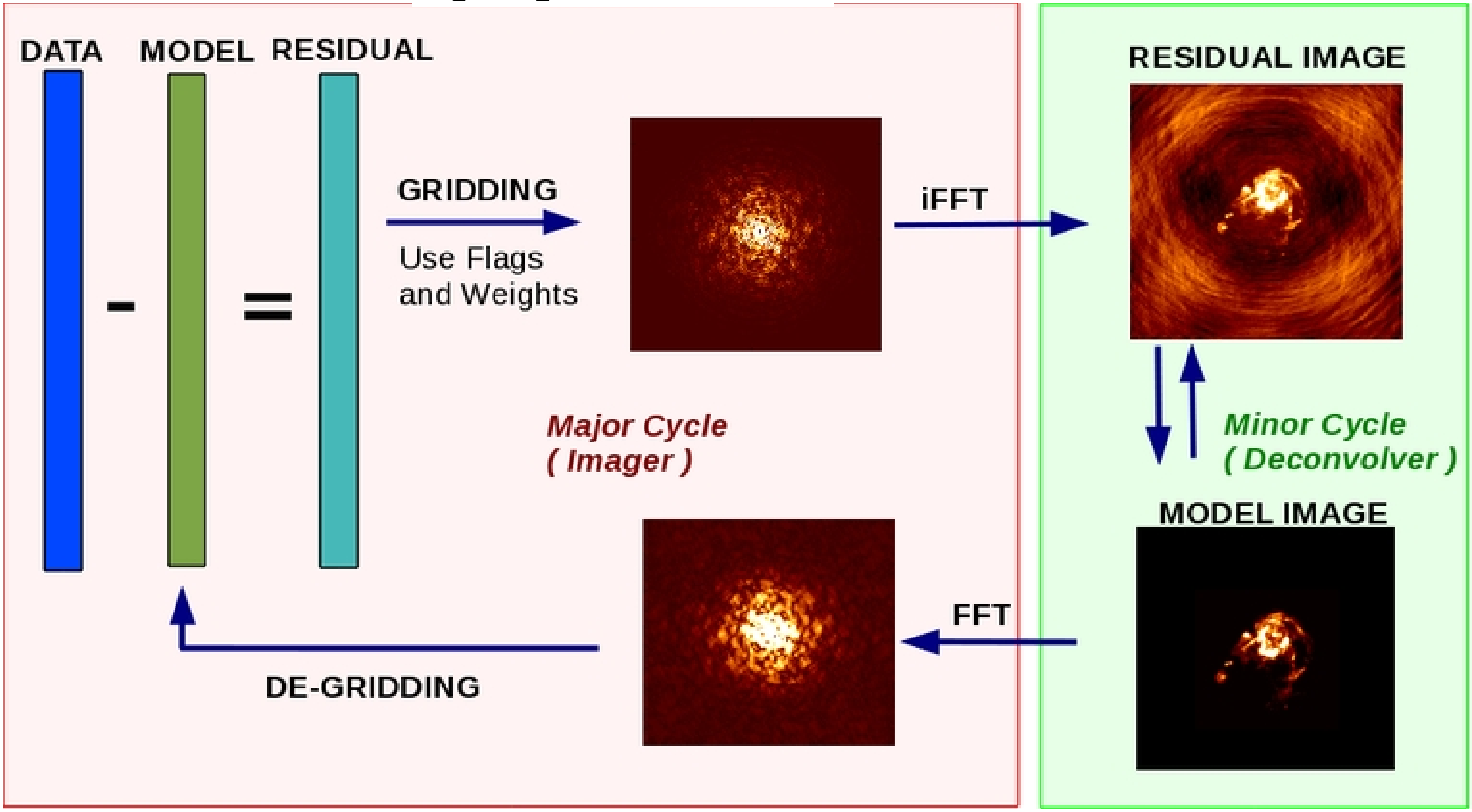


CS-Clean,
Hogbom, MT
MFS,
Multiscale

Standard, Mosaic, W-projection, A-Projection
AW-projection

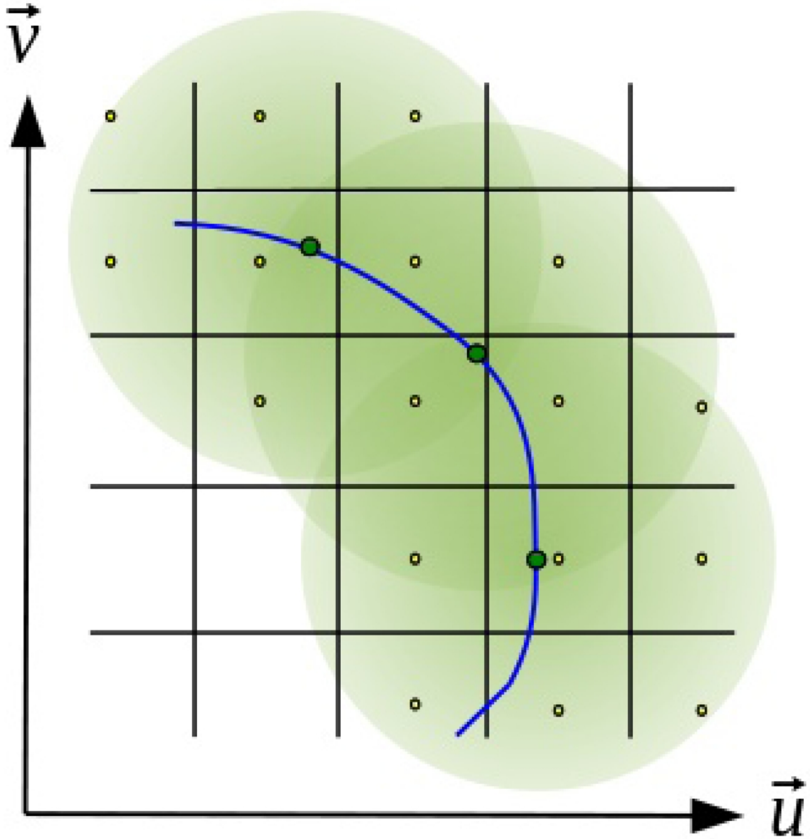
Imaging – Iterative Chi square minimization

$$[A]I^m = V^{obs}$$

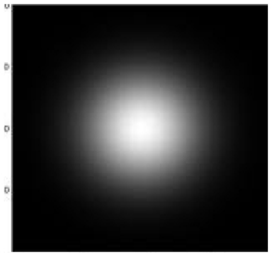


$$I^m_{i+1} = I^m_i + g[A^T W A]^+ \left(A^T W \left(V^{obs} - A I^m_i\right)\right)$$

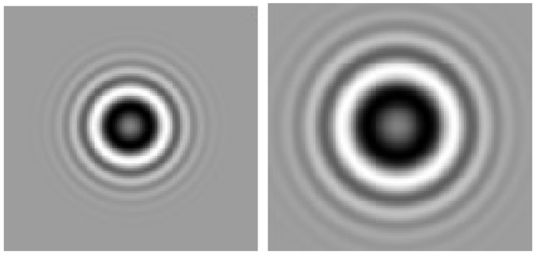
Convolutional Resampling - Gridding



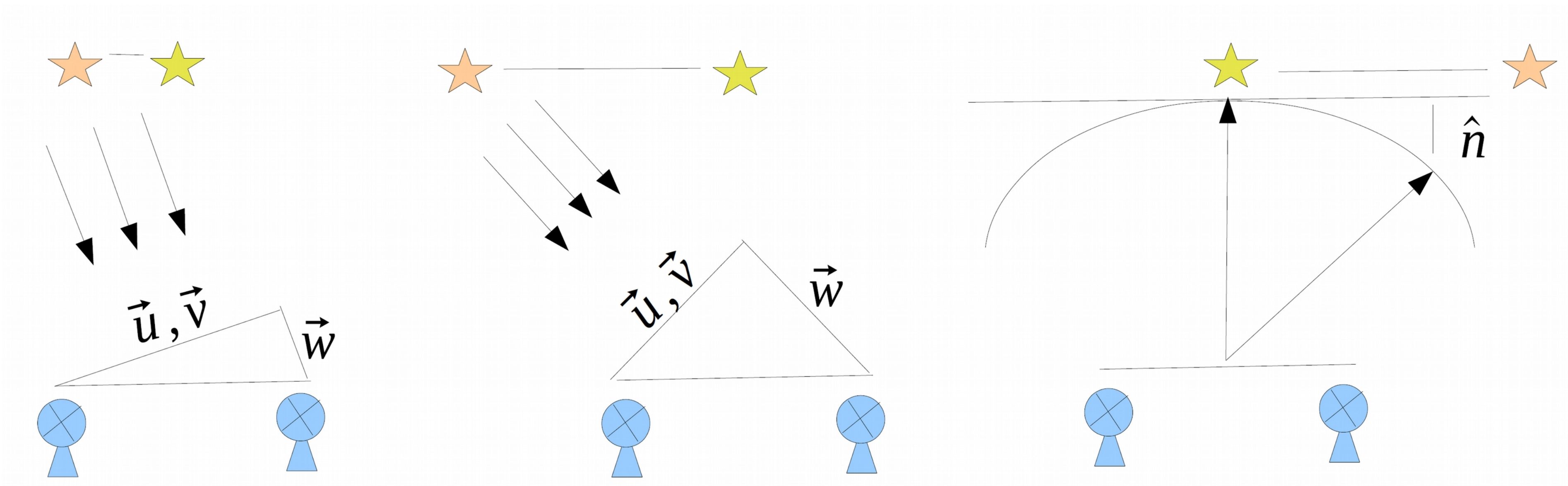
Prolate Spheroidal Function



Fresnel Kernel W-Projection



Wide-field Imaging



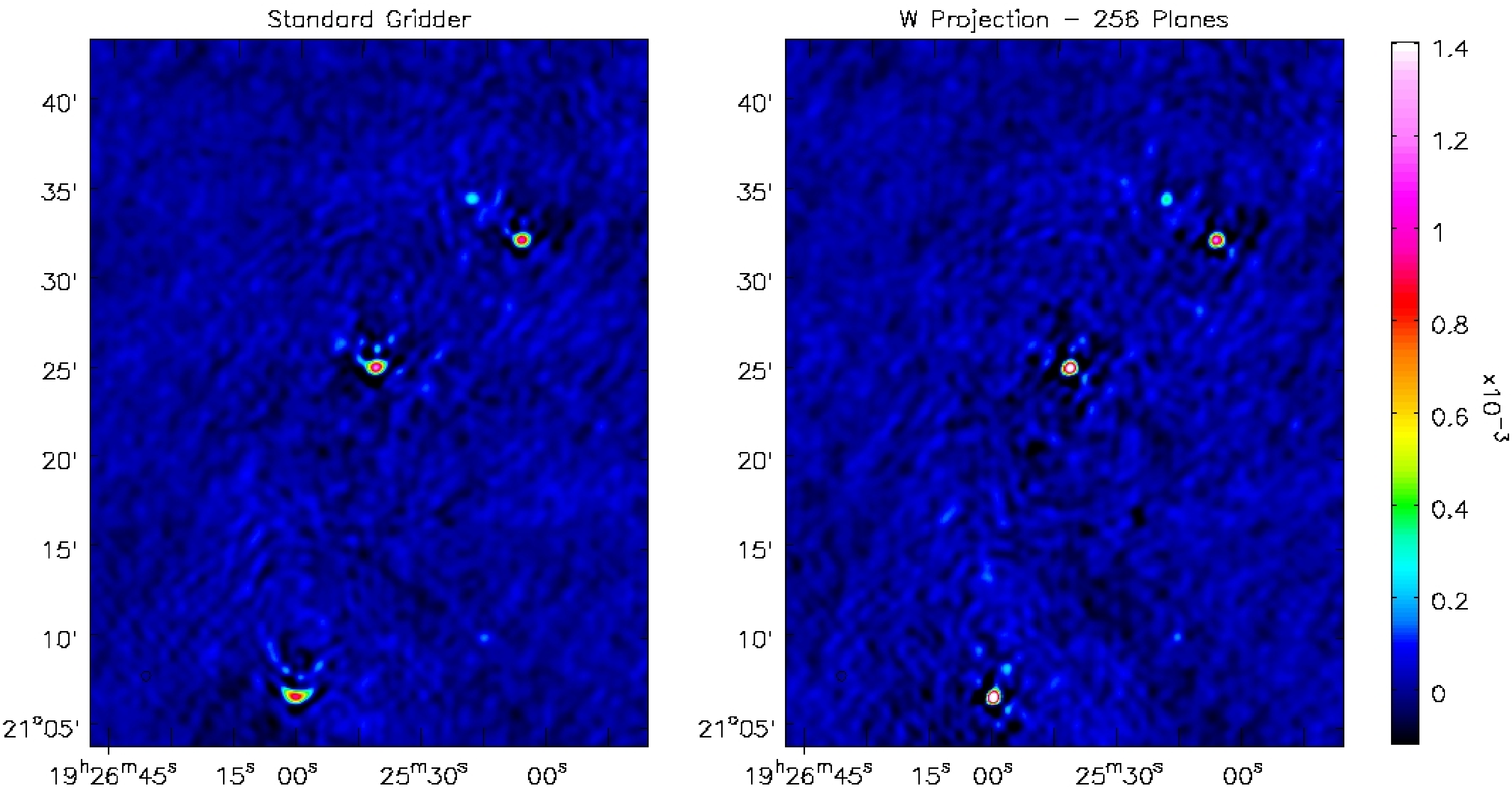
Small FoV

- w-value per baseline is small.
- vCZ holds.

Large FoV or Non-Coplanar Array

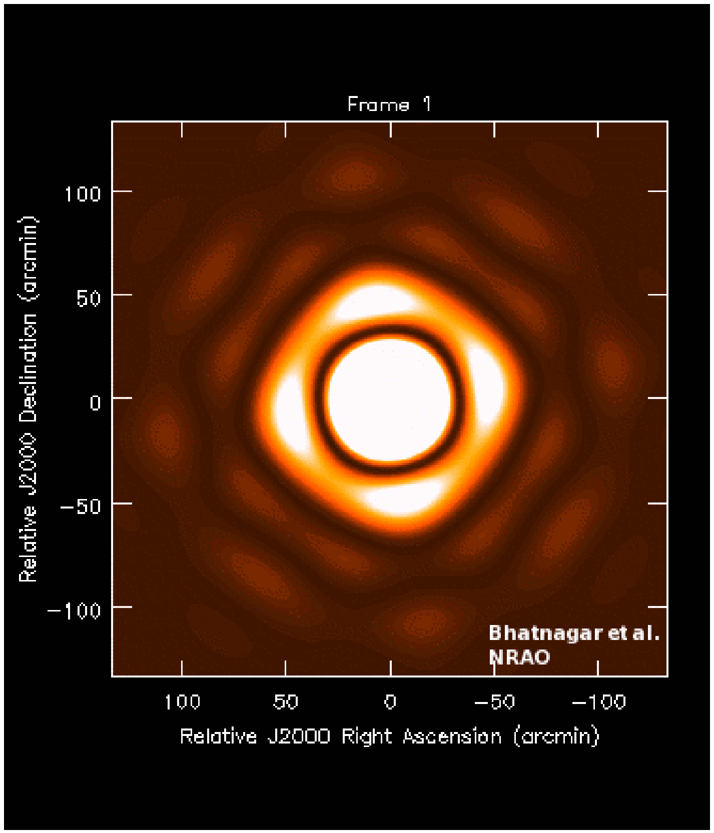
- Large w-values.
- vCZ no longer holds.
- Need to account for w in imaging
- Algorithms – facetting, w-projection

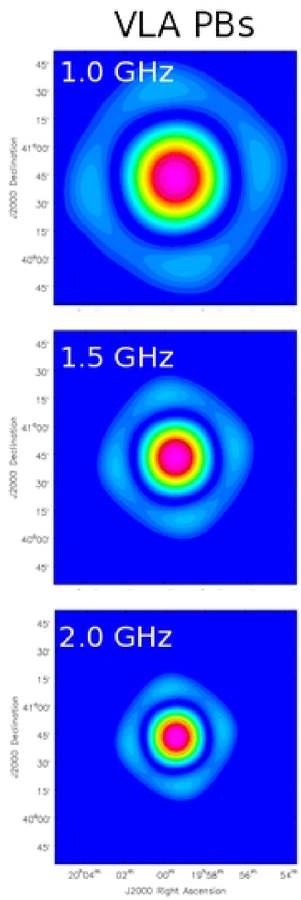
W - Projection

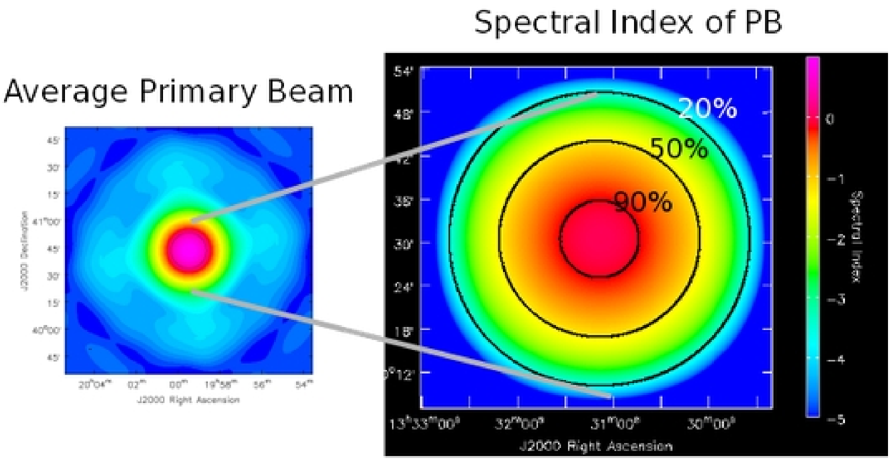


Antenna Primary Beam

Varies with time, frequency and direction on the sky





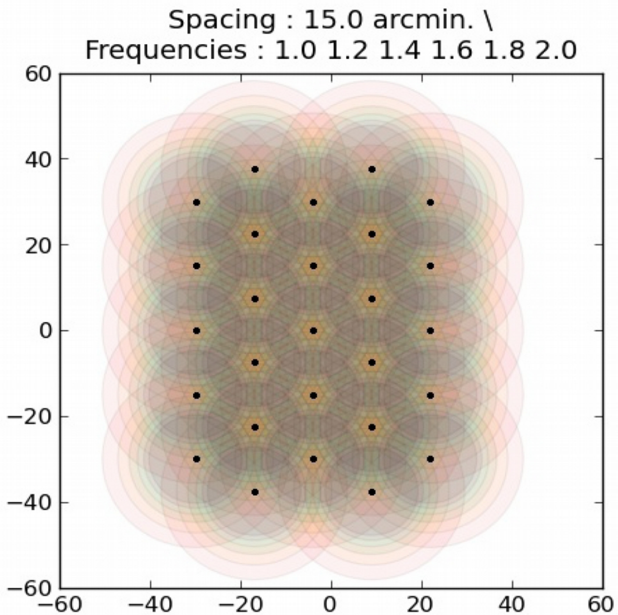


Courtesy: S. Bhatnagar, U.Rau

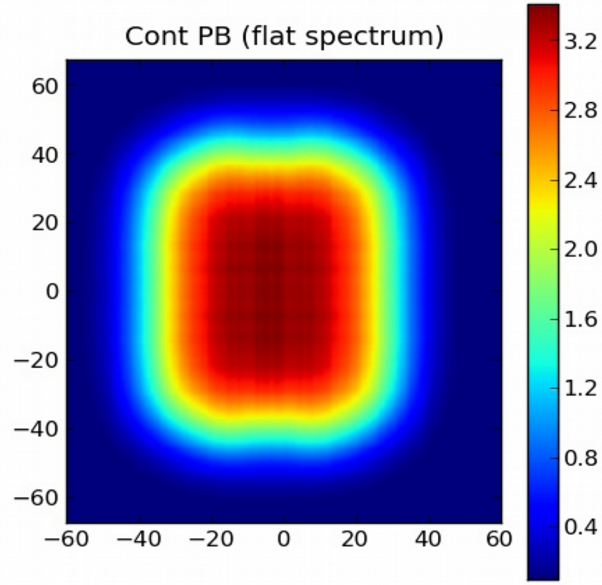
Time

Frequency

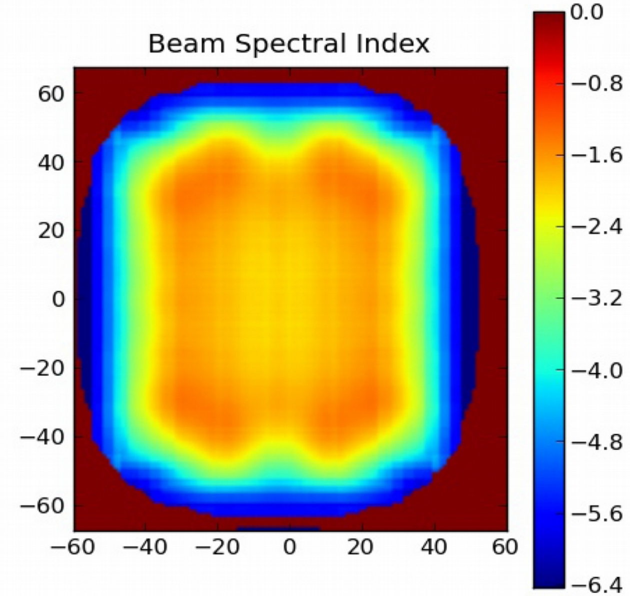
Antenna Primary Beam - Mosaic



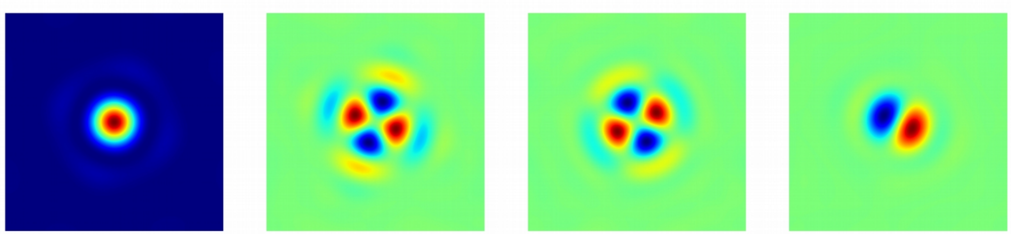
Mosaic Pointings



Continuum Sensitivity



Mosaic Spectral Index

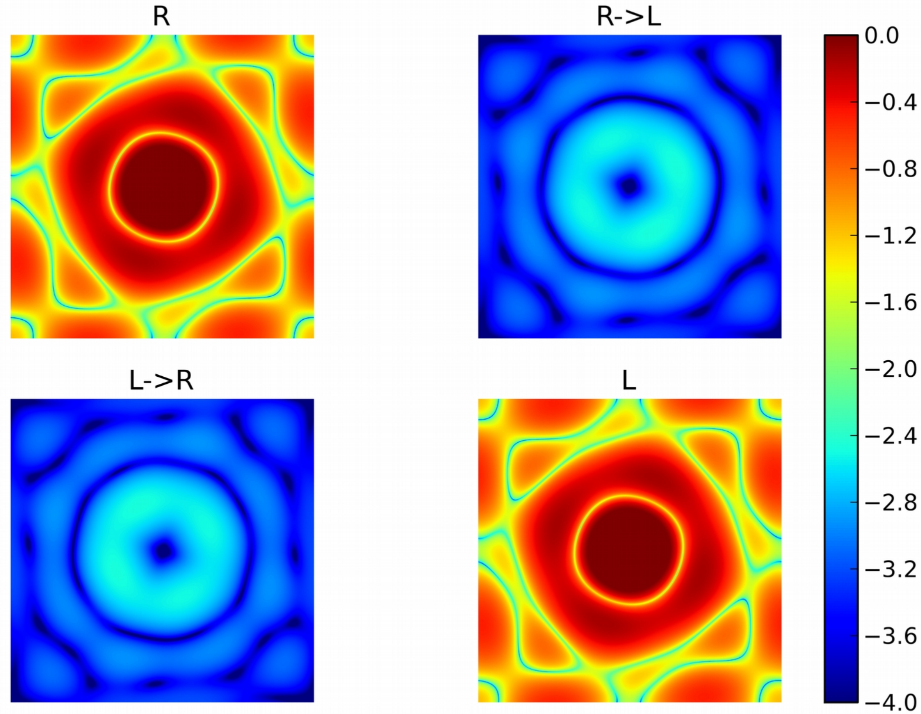


Polarized PB

Measurement Equation – Matrix Form

$$V_{ij}^{Obs} = (J_i \circledast J_j^*) \star V_{ij}^{Sky} = A_{ij} \star V_{ij}^{Sky}$$

$$F[J_i] = PB_i$$

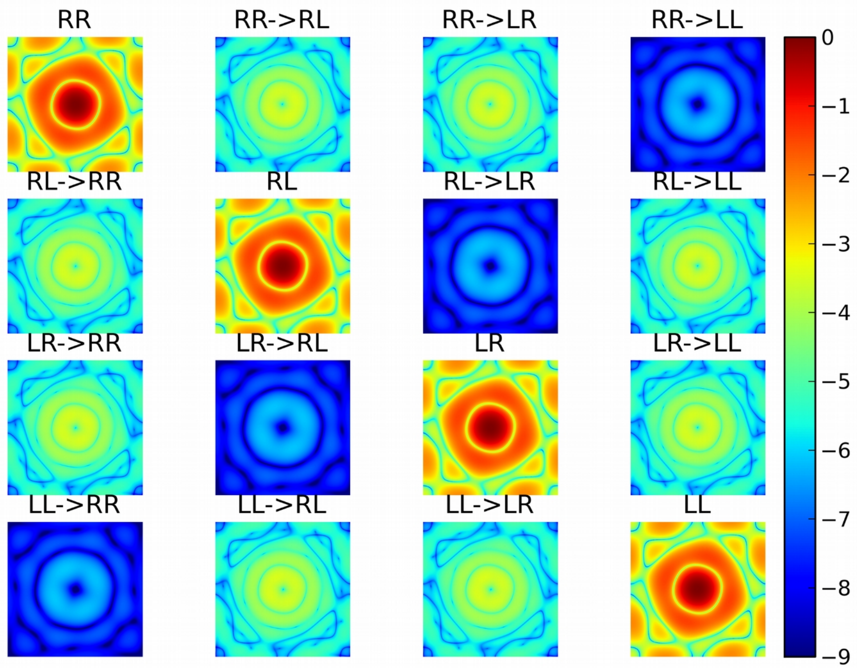


The diagonal is the antenna response function to the two orthogonal polarizations of EM waves. Off diagonal, gives the amount of cross talk

Measurement Equation – Matrix Form

$$V_{ij}^{Obs} = (J_i \circledast J_j^*) \star V_{ij}^{Sky} = A_{ij} \star V_{ij}^{Sky}$$

$$\begin{pmatrix} J_i^R J_j^{R*} & -J_i^R J_j^{RL*} & -J_i^{RL} J_j^{R*} & J_i^{RL} J_j^{RL*} \\ J_i^R J_j^{LR*} & J_i^R J_j^{L*} & -J_i^{RL} J_j^{LR*} & -J_i^{RL} J_j^{L*} \\ J_i^{LR} J_j^{R*} & -J_i^{LR} J_j^{RL*} & J_i^L J_j^{R*} & -J_i^L J_j^{RL*} \\ J_i^{LR} J_j^{LR*} & J_i^{LR} J_j^{L*} & J_i^L J_j^{LR*} & J_i^L J_j^{L*} \end{pmatrix}$$

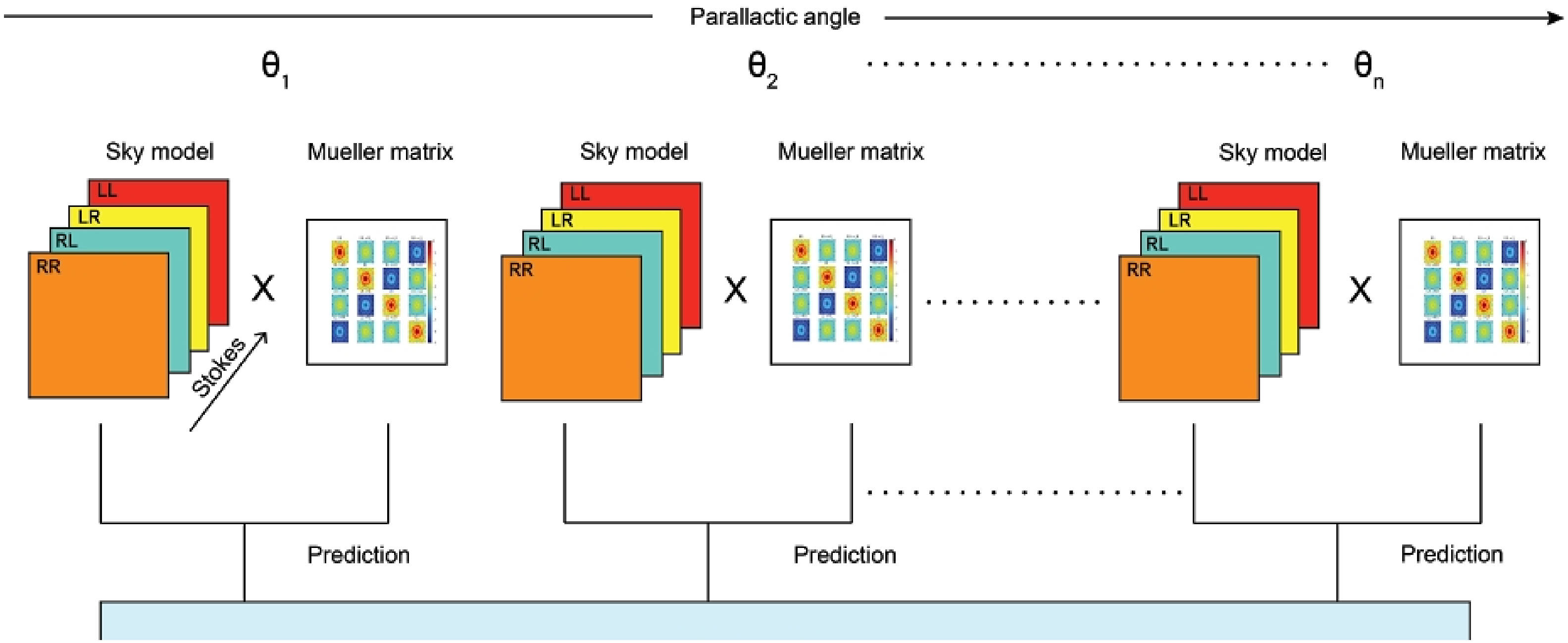


The cross correlation of two antenna jones matrices.

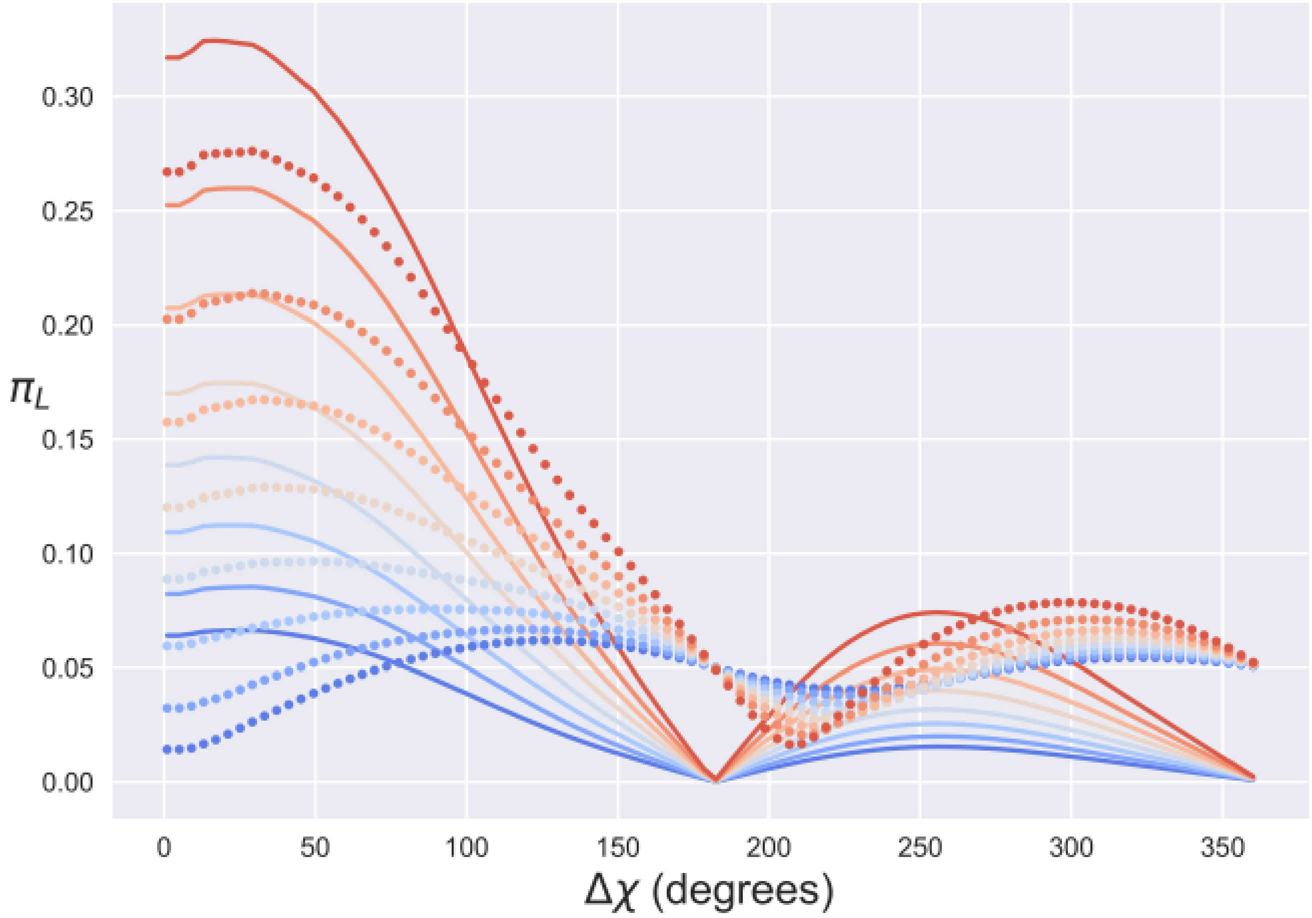
In data domain the matrix is unitary.

Simulations – Effect of Mueller Matrix

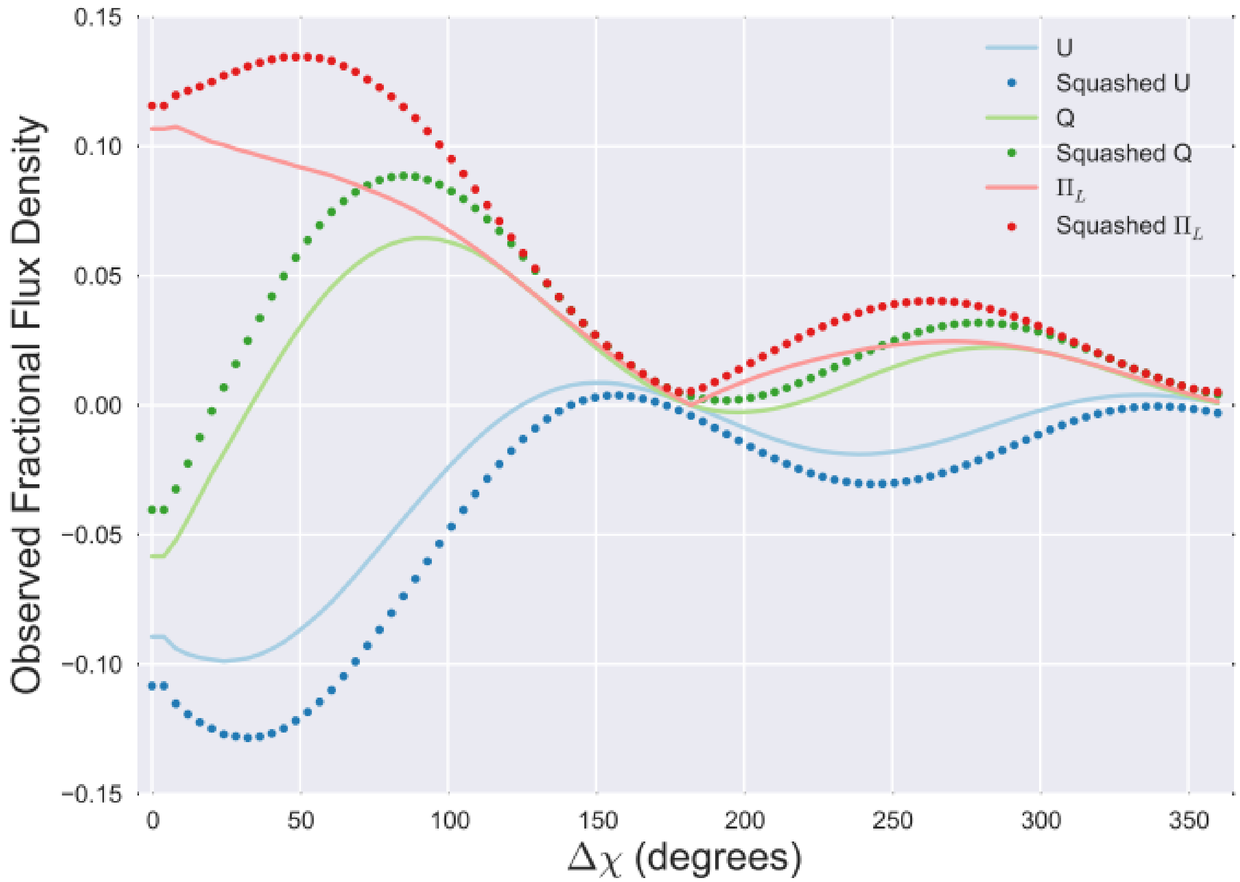
- A single point source at HPBW a) unpolarized b) 10% polarized.
- Simulations spanned -4 to +4 hours in hour angle, L-Band, 8 spws, 1GHz effectiv
- Simulation external to the CASA framework in python, imaging in CASA.
- Quantities of interest, Stokes Q & U, Polarized Intensity, RMSF.



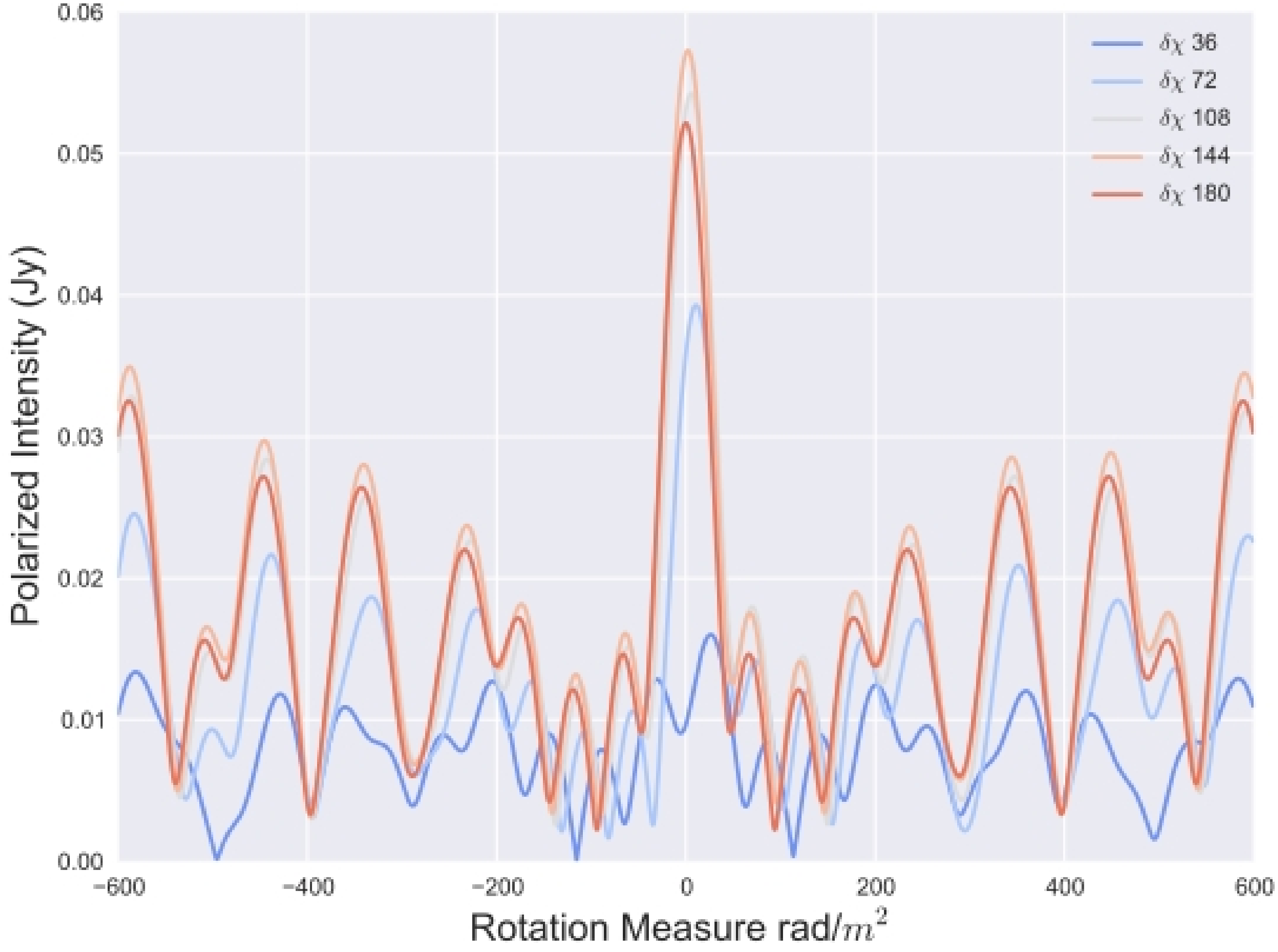
Polarization Leakage - I



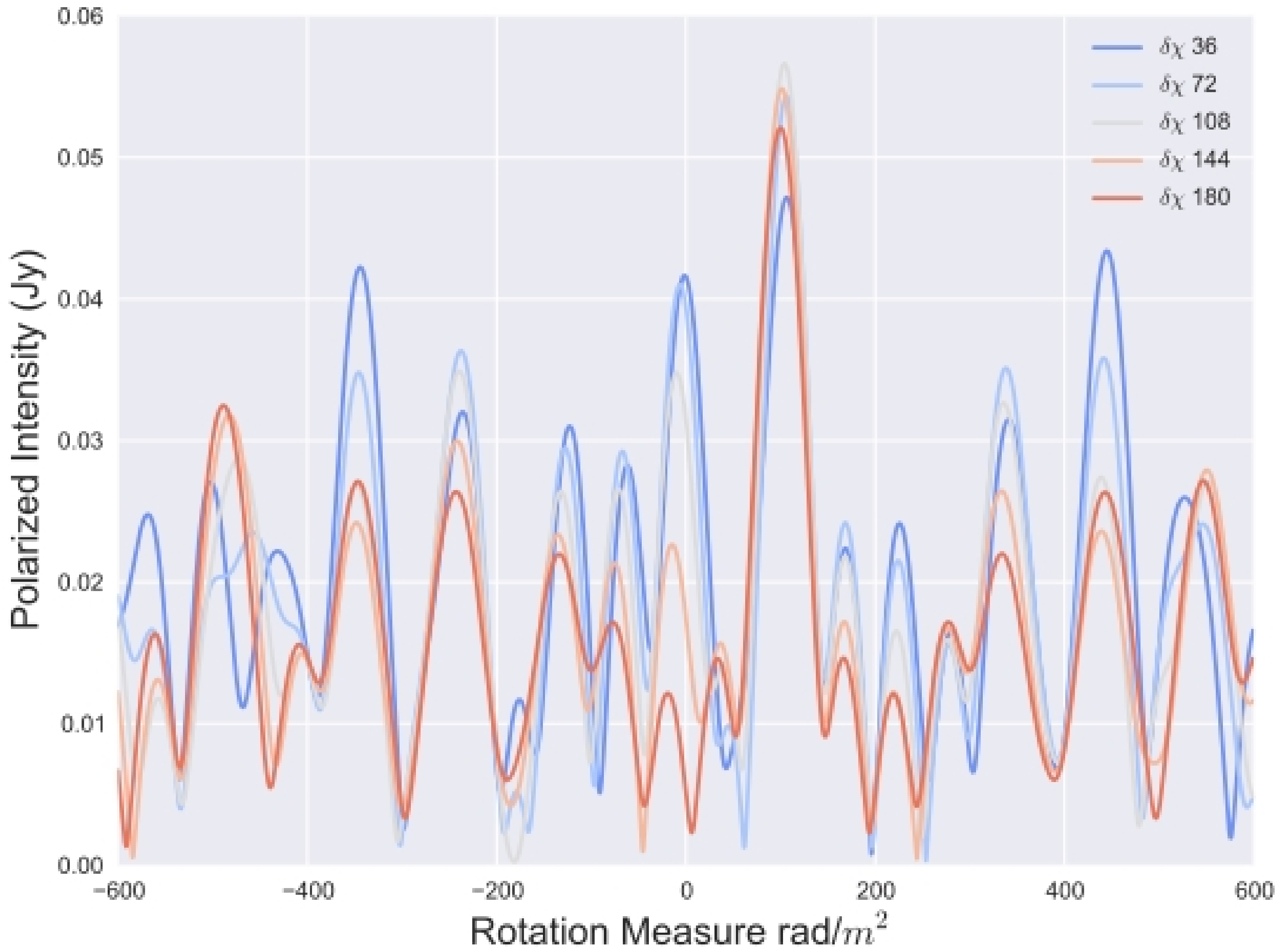
Polarization Leakage - II



Polarization Leakage – III – RMSF (RM=0)



Polarization Leakage – IV – RMSF (RM=100)



FM AW-Projection -I

$$\vec{V}_k^{Obs} = (A_k^\circ \star \vec{V}^\circ)\delta_k \qquad k = ij, \nu, t$$

Observed Visbilities

Continuous Coherence Function

Sampling Function

$$A_k^{\circ\,-1} = \frac{adj(A_k^\circ)}{det(A_k^\circ)}$$

Correction Matrix is applied in two steps.

FM AW-Projection - II

Gridding with the hermitian conjugate transpose of A

$$\vec{V}_k^M = \sum_k A^{M^\dagger} \star (A_k^\circ \star \vec{V}^\circ)\delta_k$$

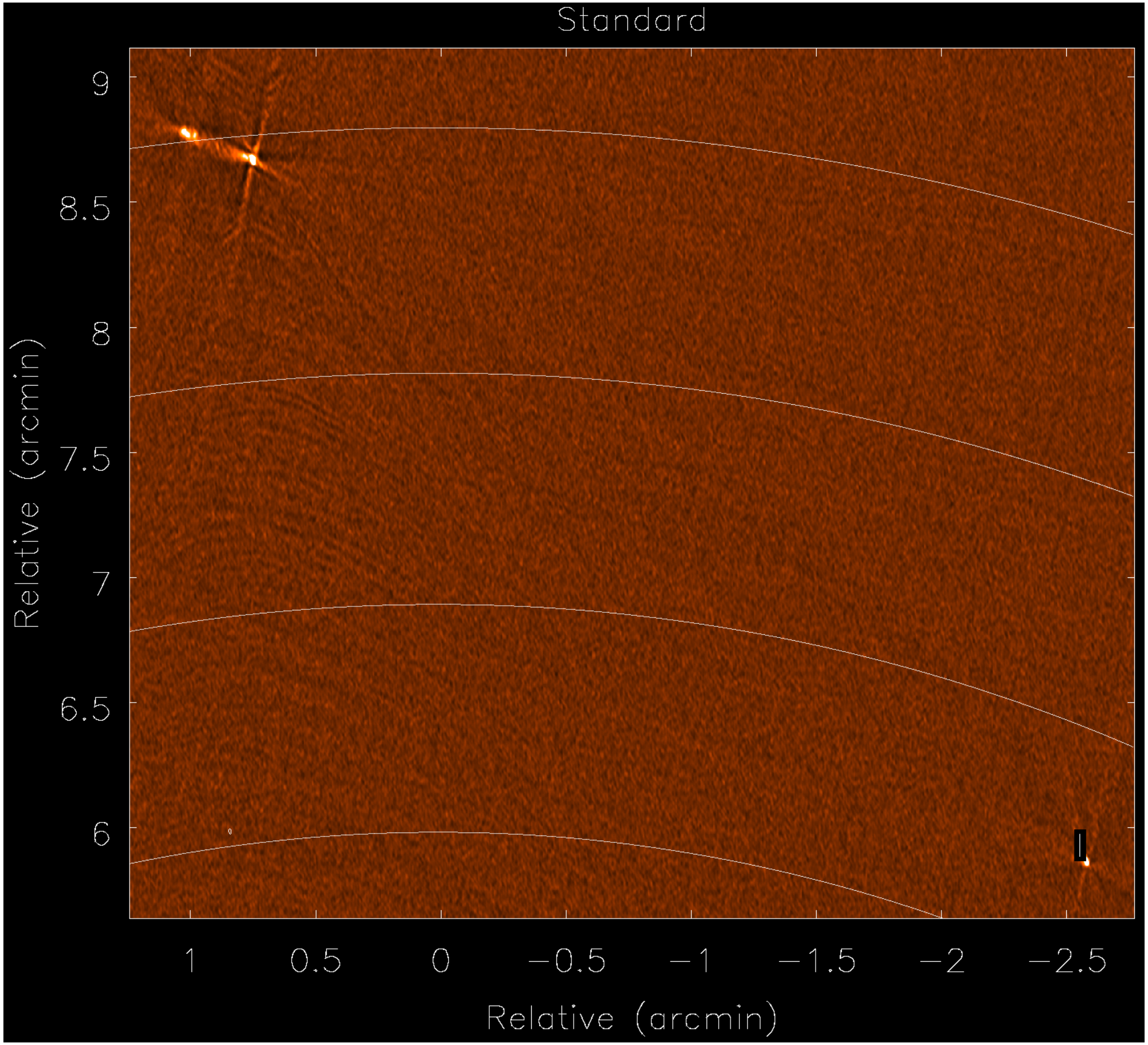
Accumulating the averaged convolution function for normalization

$$\overline{A^M} = \sum_k A_k^M$$

Normalized Residual Image - End of Major cycle

$$\vec{I}^R = \frac{F \sum_k \vec{V}_k^{Obs} - \vec{V}_k^M}{det(F\overline{A^M})}$$

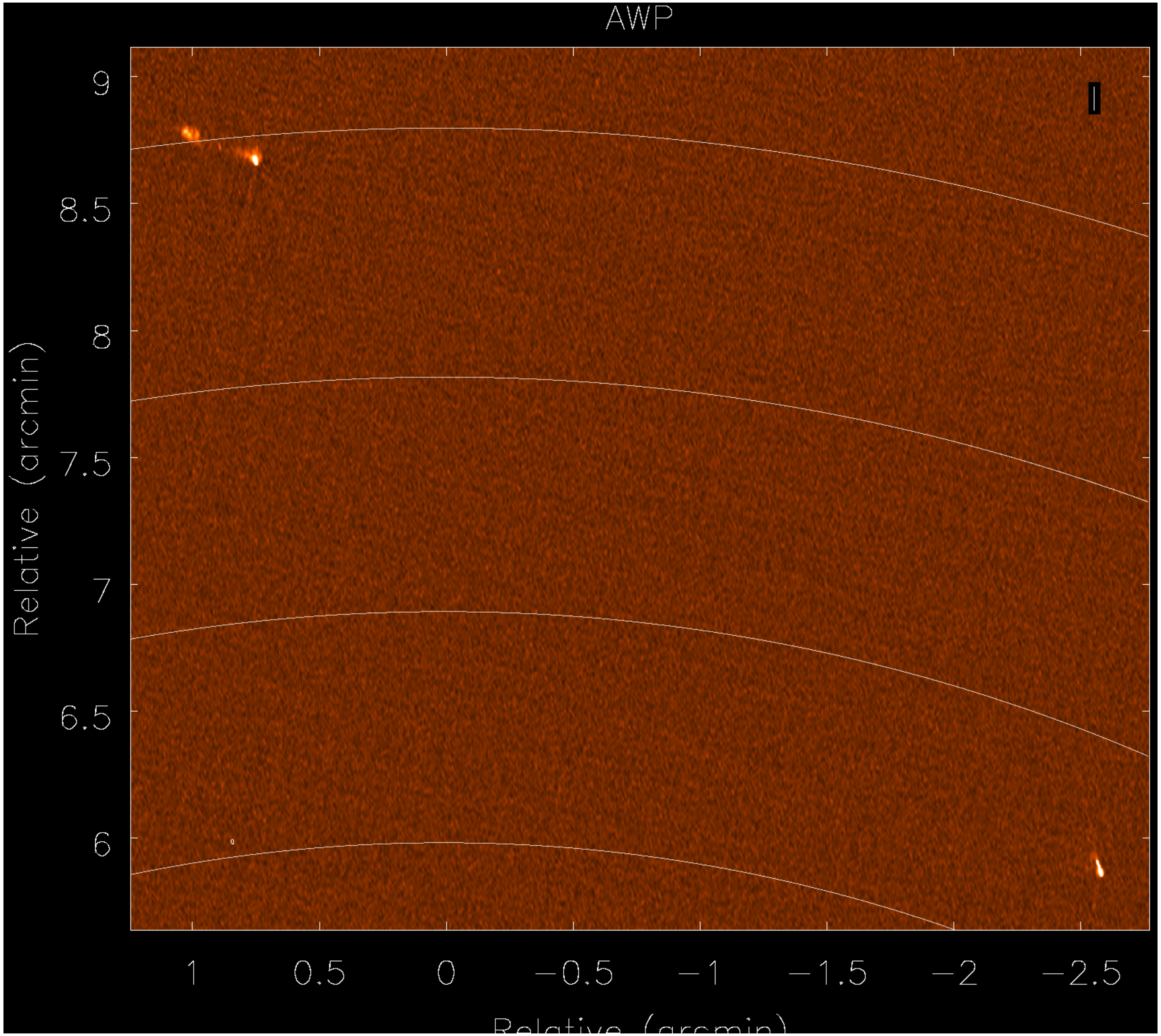
Standard Gridder : HUDF S-Band A-configuration



0.4 microJy

R.M.S

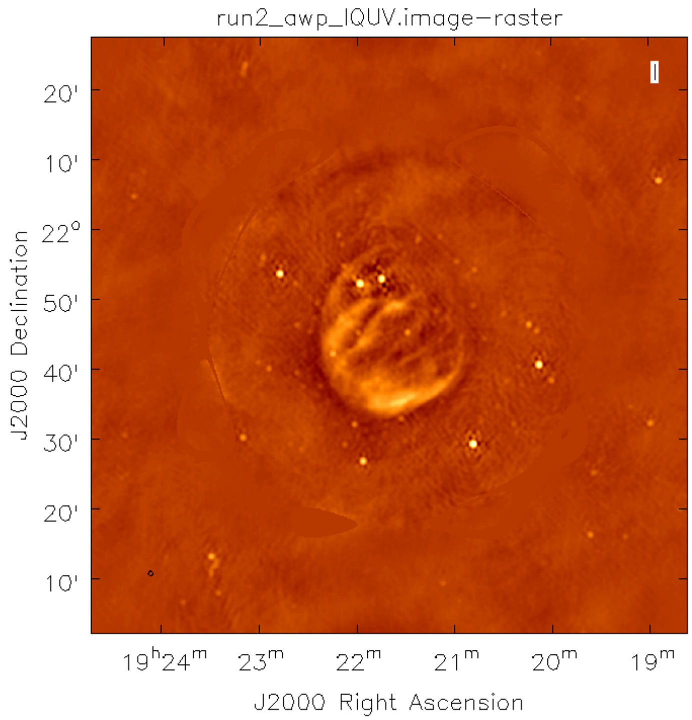
A – Projection : HUDF S-Band A-configuration



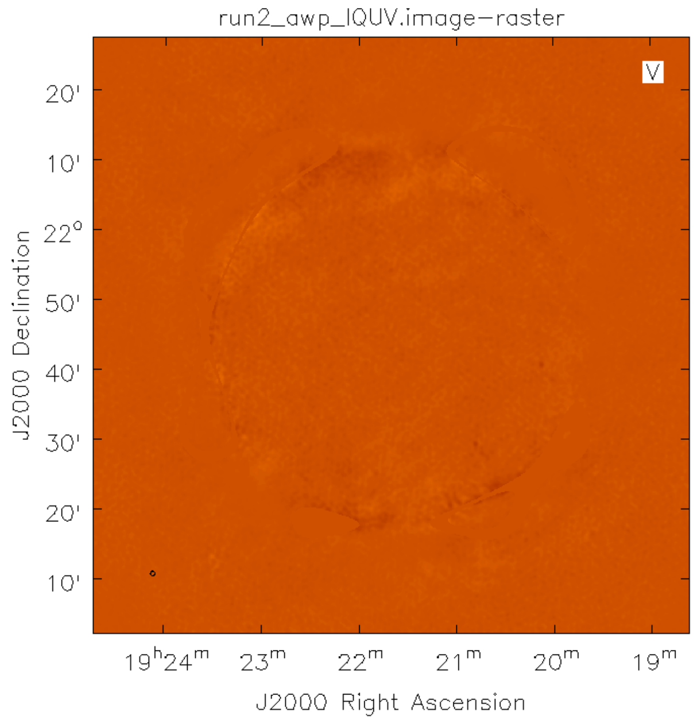
0.4 microJy

R.M.S

FM AW-Projection – III

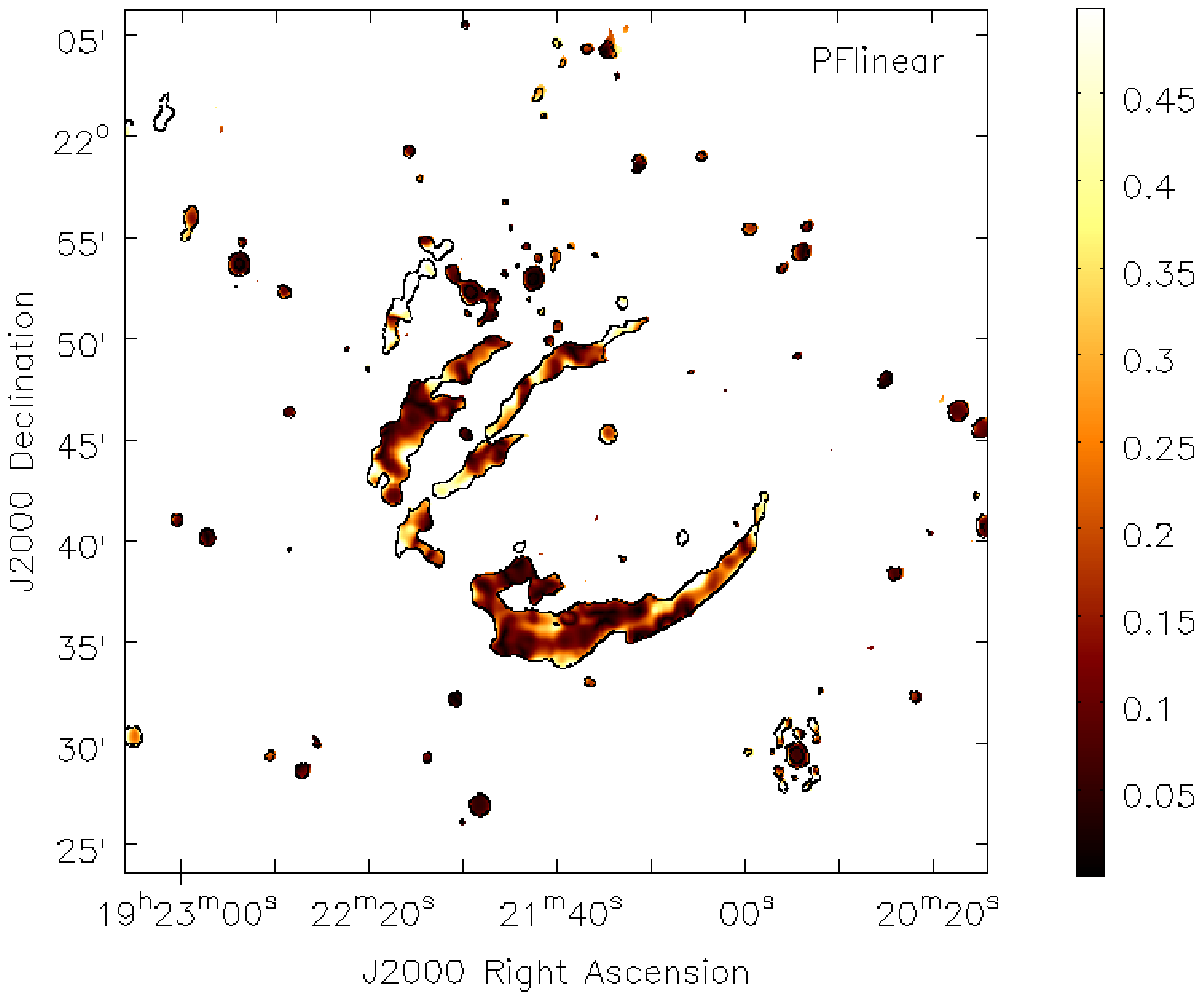


4.4 microJy



4 microJy theoretical sensitivity

FM AW-Projection – IV – Polarized Intensity



Who needs FM AW-Projection

1. What fraction of the field of view contains your source of interest
   - 50% in power and beyond at reference frequency

2. What is your bandwidth ratio ?
   - Bandwidth/(Reference Frequency) > 0.2

3. Did you make a pointed mosaic for sensitvity ?
   - Narrow band mosaic ?
   - Broad band mosaic ?

4. Did you integrate for a very long time for increased senstivity ?

5. What fidelity do you require in your source flux density and spectral index ?

6. Do you need wide-field polarization leakage correction ?

Conclusions

- Full Mueller AWP required for wide field Polarimetry. A requirement for most deep surveys with current and future radio interferometric arrays.
- In the absence of FM AWP the beam leaks flux from Stokes I to Stokes Q and U and alters the true polarized signal and for short integrations reduced RM signal and P due to depolarization.
- Correction via A-Projection is computationally expensive and only feasible when employed on HPC infrastructure.