Fundamental Limitations on the Calibration of Redundant 21 cm Instruments

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Redundant 21 cm Cosmology Instruments

- Several recent 21 cm cosmology instruments have redundant array layouts

Hydrogen Epoch of Reionization Array (HERA)

Image Source: HERA

Precision Array for Probing the EoR (PAPER)

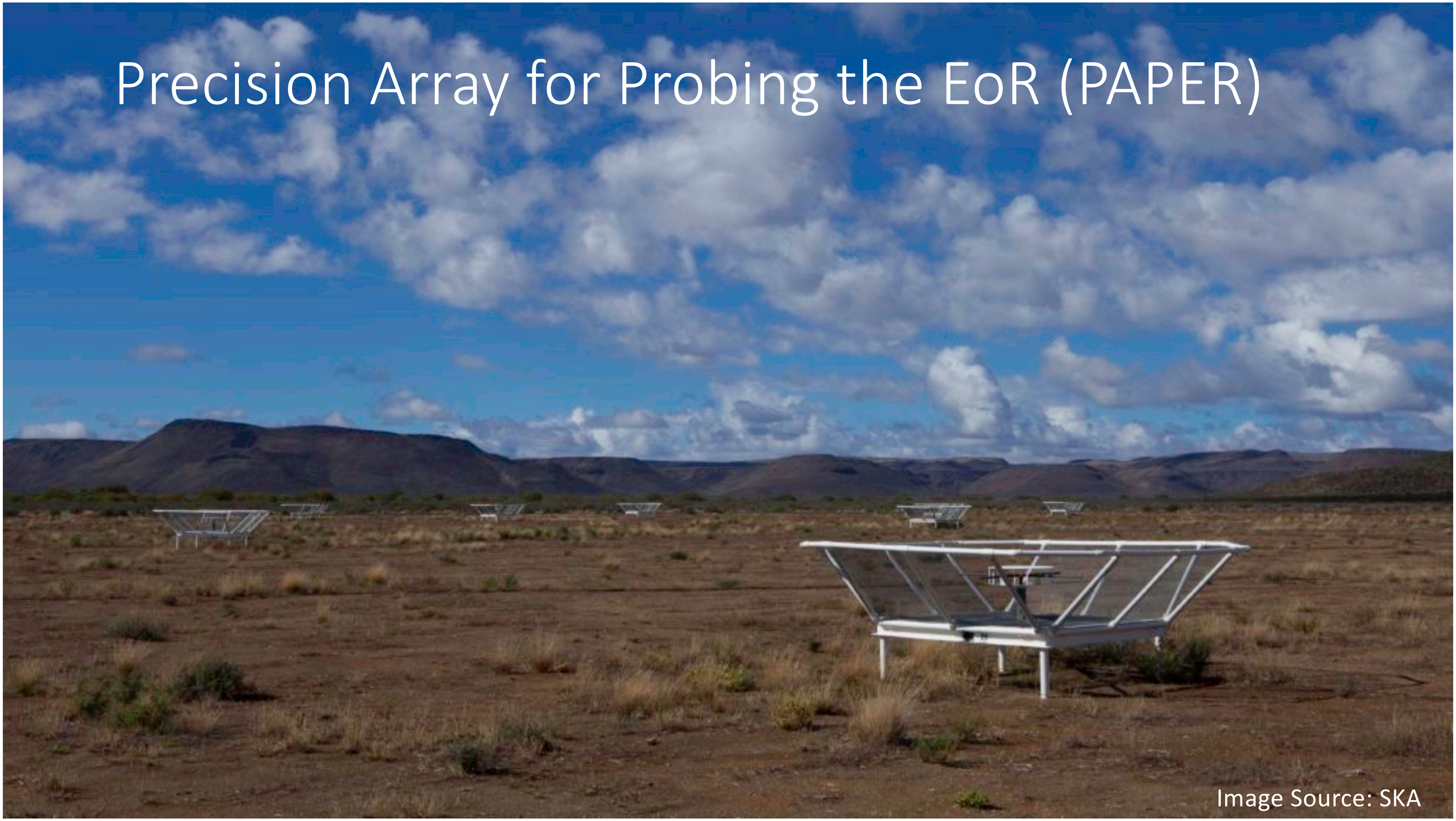
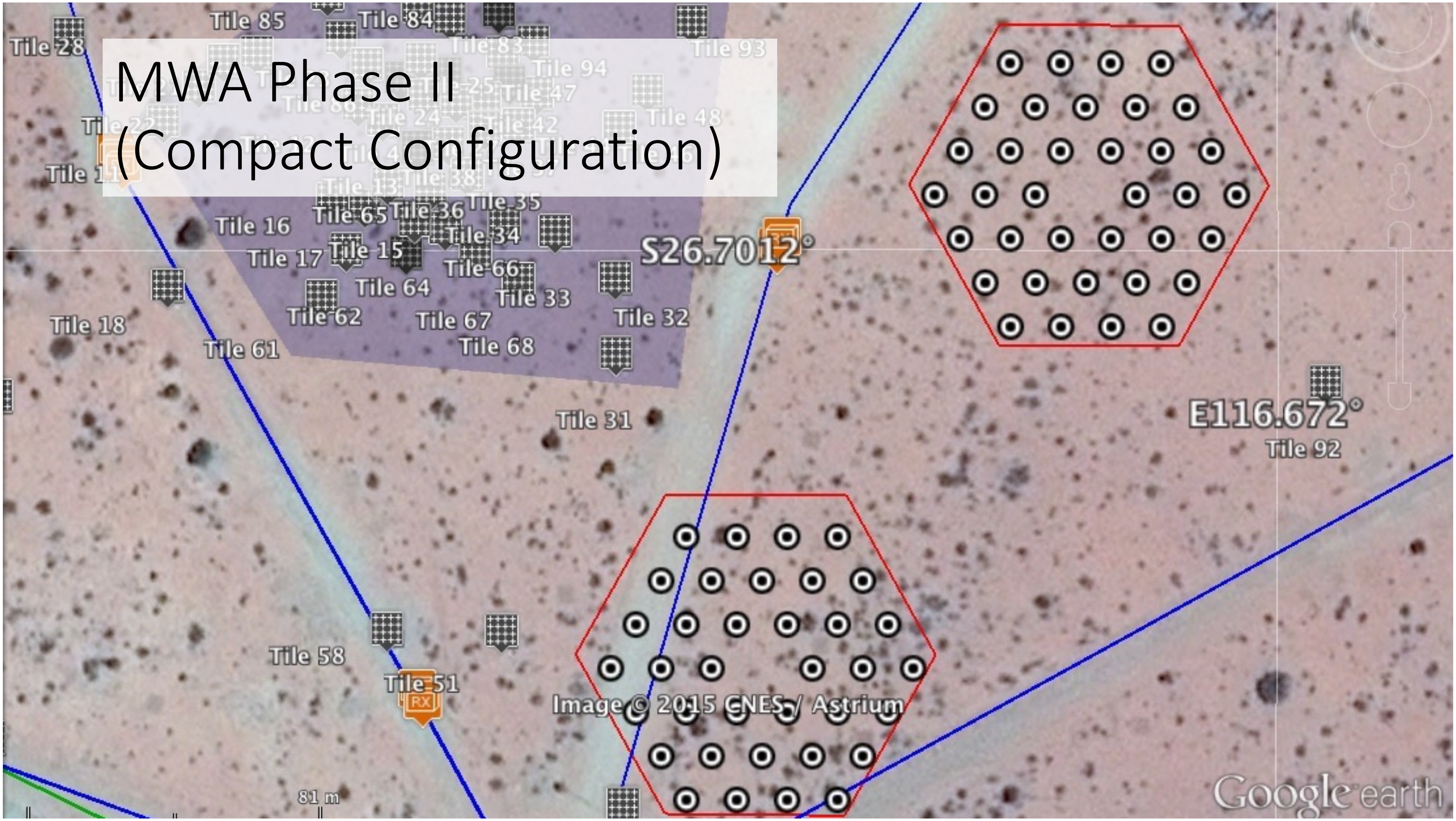


Image Source: SKA

MWA Phase II
(Compact Configuration)



Redundant 21 cm Cosmology Instruments

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- Better power spectrum sensitivity

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- Better power spectrum sensitivity
- Enable redundant calibration

Redundant 21 cm Cosmology Instruments

- Several recent 21 cm cosmology instruments have redundant array layouts
- Better power spectrum sensitivity
- Enable **redundant calibration**

The Measurement Equation

$$v_{ij}(f) = g_i(f) g_j^*(f) u_{ij}(f) + n_{ij}(f)$$

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antenna gains

"true" visibility

The Measurement Equation

$$v_{ij}(f) = g_i(f) g_j^*(f) u_{ij}(f) + n_{ij}(f)$$

measured visibility

antenna gains

"true" visibility

noise

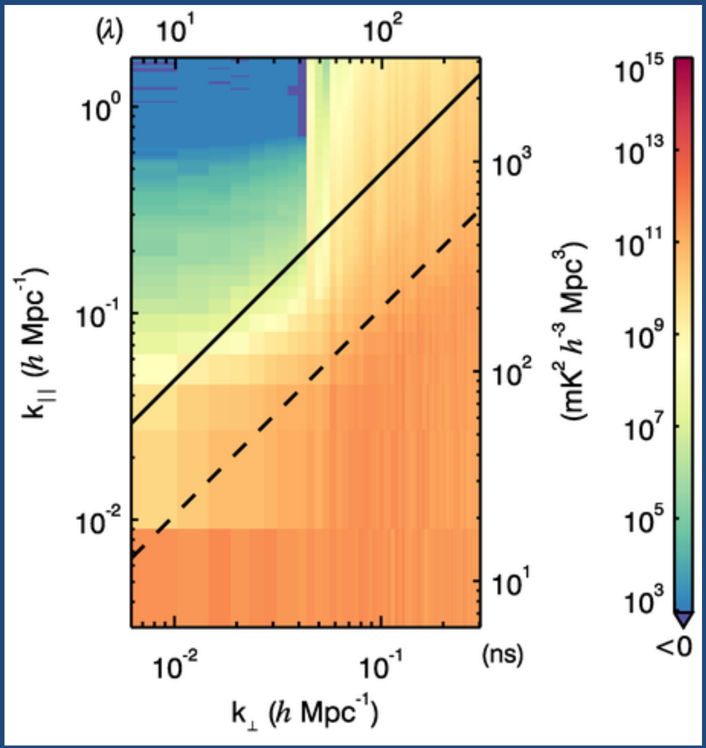
Sky-Based Calibration

- Use sky and instrument models to approximate the true visibilities

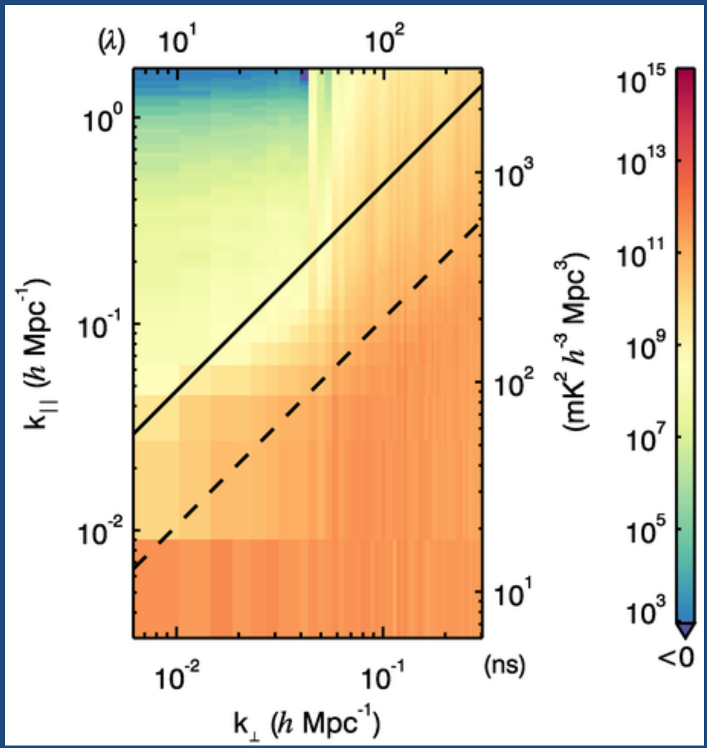
$$v_{ij}(f) = g_i(f)g_j^*(f)\overset{m_{ij}(f)}{\cancel{u_{ij}(f)}} + n_{ij}(f)$$

Calibration errors from an incomplete sky model contaminate the EoR signal (Barry et al. 2016)

Perfect calibration



Calibrated to 4,000 of 51,821 sources



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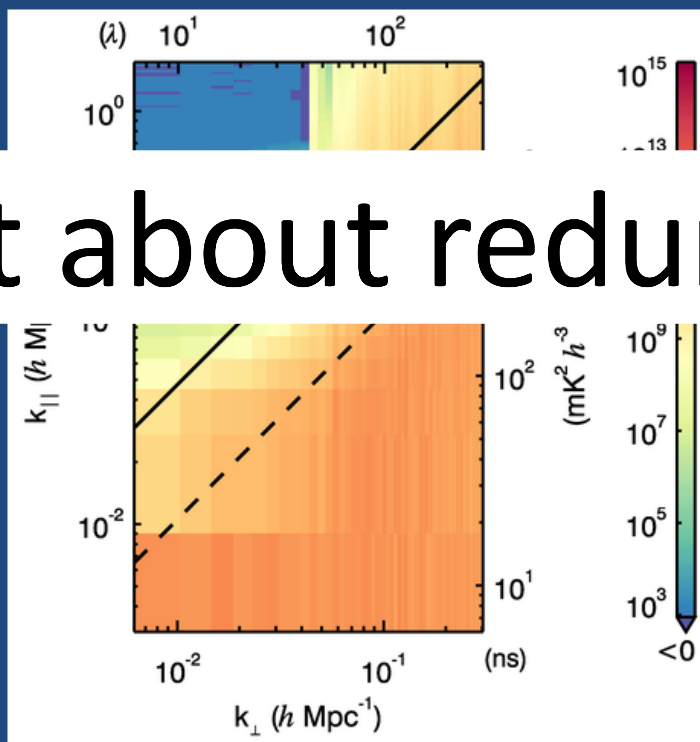

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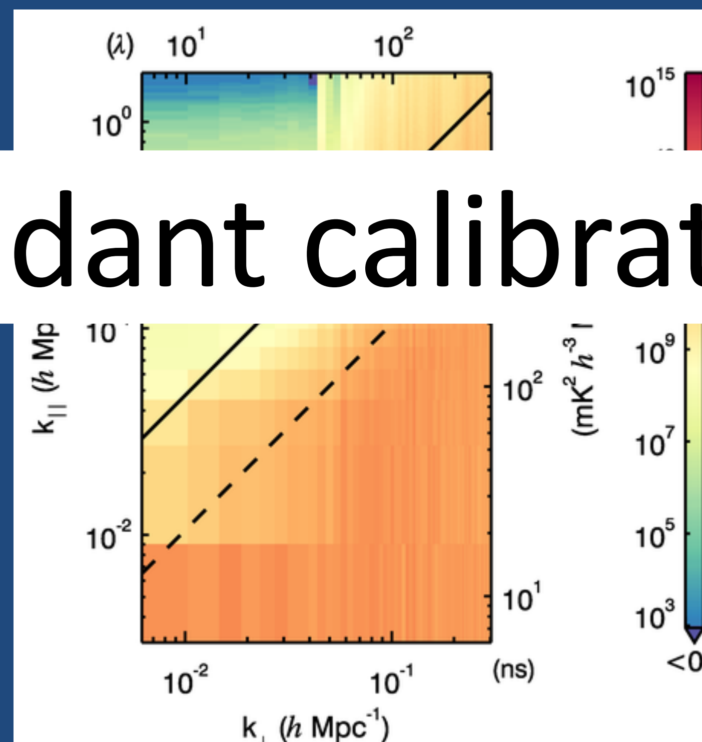
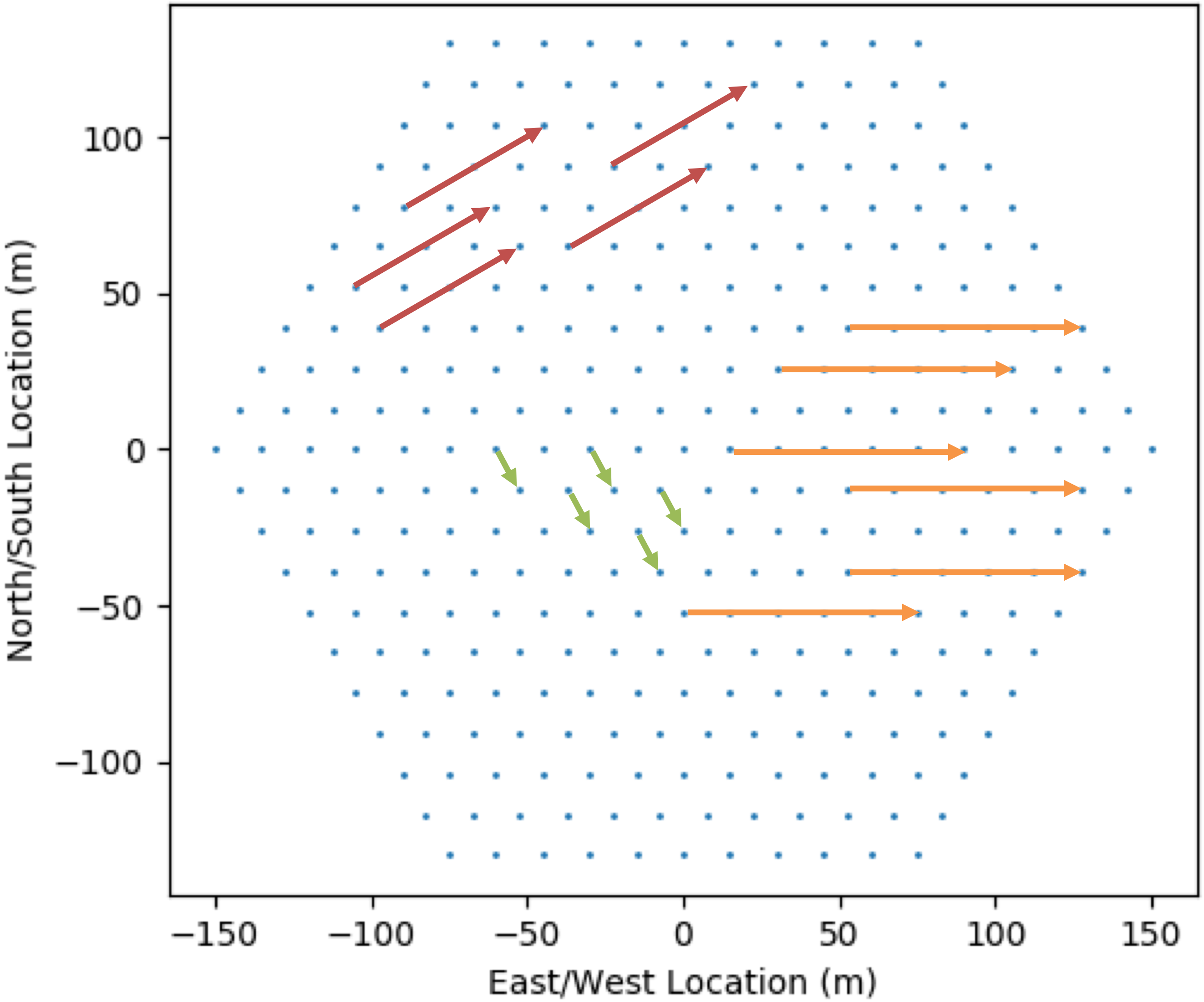


what about redundant calibration?



$$u_{1\,2} = u_{3\,4} = u_{5\,6} = \ldots$$

$$u_{1\,7} = u_{3\,9} = u_{5\,11} = \ldots$$

$$u_{2\,50} = u_{3\,51} = u_{4\,52} = \ldots$$

Redundant Calibration

- Assume that redundant baselines measure the same visibilities

$$v_{ij}(f) = g_i(f) g_j^*(f) \overset{u_\alpha(f)}{\cancel{u_{ij}(f)}} + n_{ij}(f)$$

Redundant Calibration - Degeneracies

$$v_{ij}(f) = g_i(f)g_j^*(f)u_\alpha(f) + n_{ij}(f)$$

- Amplitude degeneracy: $g_i \to Ag_i, u_\alpha \to A^{-2}u_\alpha$

Redundant Calibration - Degeneracies

$$v_{ij}(f) = g_i(f)g_j^*(f)u_\alpha(f) + n_{ij}(f)$$

- Amplitude degeneracy: $g_i \rightarrow Ag_i, u_\alpha \rightarrow A^{-2}u_\alpha$
- Phase degeneracy: $g_i = |g_i|e^{i\phi_i} \rightarrow |g_i|e^{i(\phi_i+\Delta)}$

Redundant Calibration - Degeneracies

$$v_{ij}(f) = g_i(f)g_j^*(f)u_\alpha(f) + n_{ij}(f)$$

- Amplitude degeneracy: $g_i \to Ag_i, u_\alpha \to A^{-2}u_\alpha$
- Phase degeneracy: $g_i = |g_i|e^{i\phi_i} \to |g_i|e^{i(\phi_i+\Delta)}$
- Phase gradient degeneracy: $g_i = |g_i|e^{i\phi_i} \to |g_i|e^{i(\phi_i+\Delta_x x_i+\Delta_y y_i)}$,

$$u_\alpha = |u_\alpha|e^{i\phi_\alpha} \to |u_\alpha|e^{i[\phi_\alpha+\Delta_x(x_m-x_k)+\Delta_y(x_m-x_k)]}$$

Redundant Calibration - Degeneracies

Four degenerate parameters *per frequency*

$$A(f), \Delta(f), \Delta_x(f), \Delta_y(f)$$

Redundant Calibration

Relative Calibration

- Use redundancy to calibrate up to degeneracies

Redundant Calibration

Relative Calibration

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Absolute Calibration

- Use a sky model to calculate degenerate parameters

Redundant Calibration

Relative Calibration

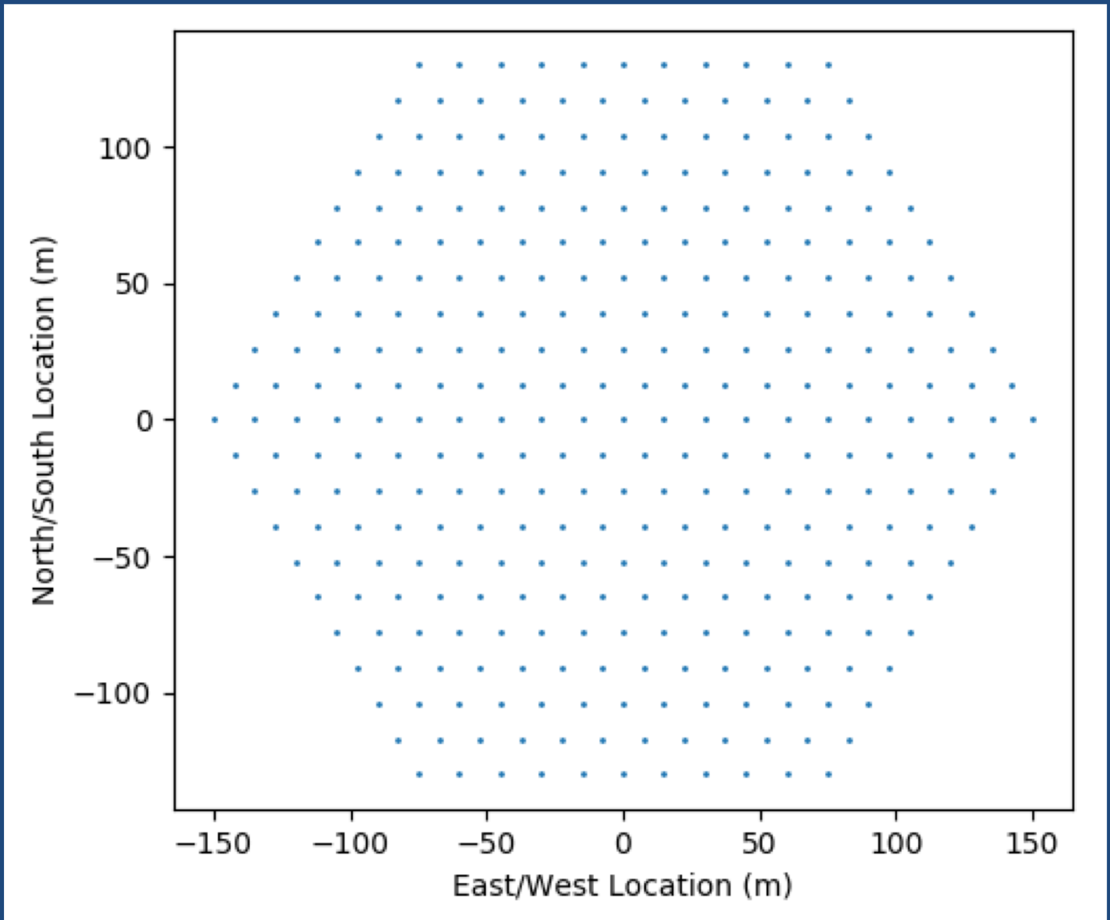
- Use redundancy to calibrate up to degeneracies

Absolute Calibration

- Use a sky model to calculate degenerate parameters

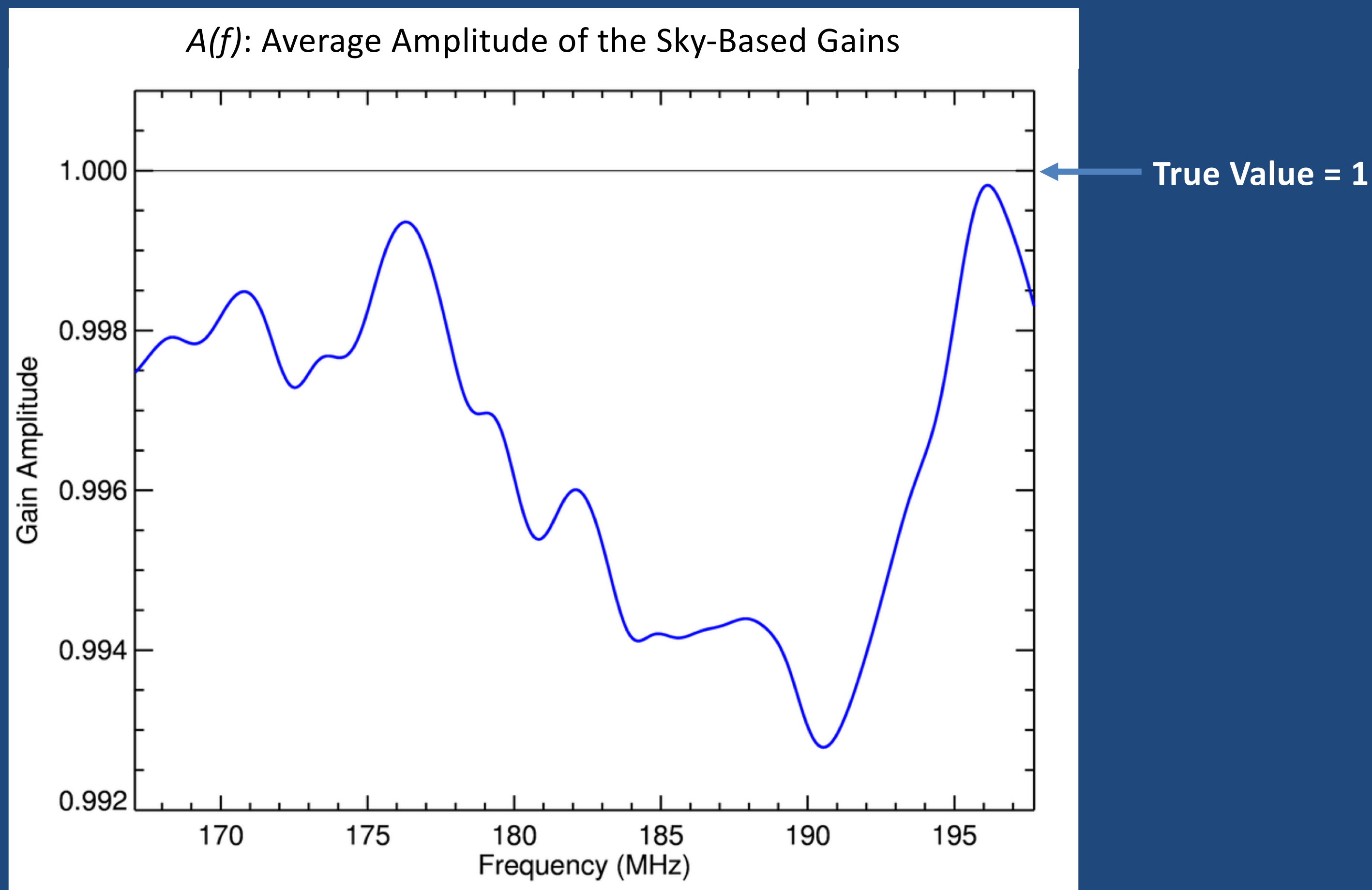
- Combine to calculate calibration solutions $g_i(f)$

Incomplete Sky Models and Absolute Calibration



- Hexagonal array with 331 antennas
- MWA beam model
- Simulate visibilities from 51,821 sources
- Absolute calibration from a sky model of 4,000 sources: calculate $A(f)$, $\Delta_x(f)$, and $\Delta_y(f)$

Incomplete Sky Models and Absolute Calibration



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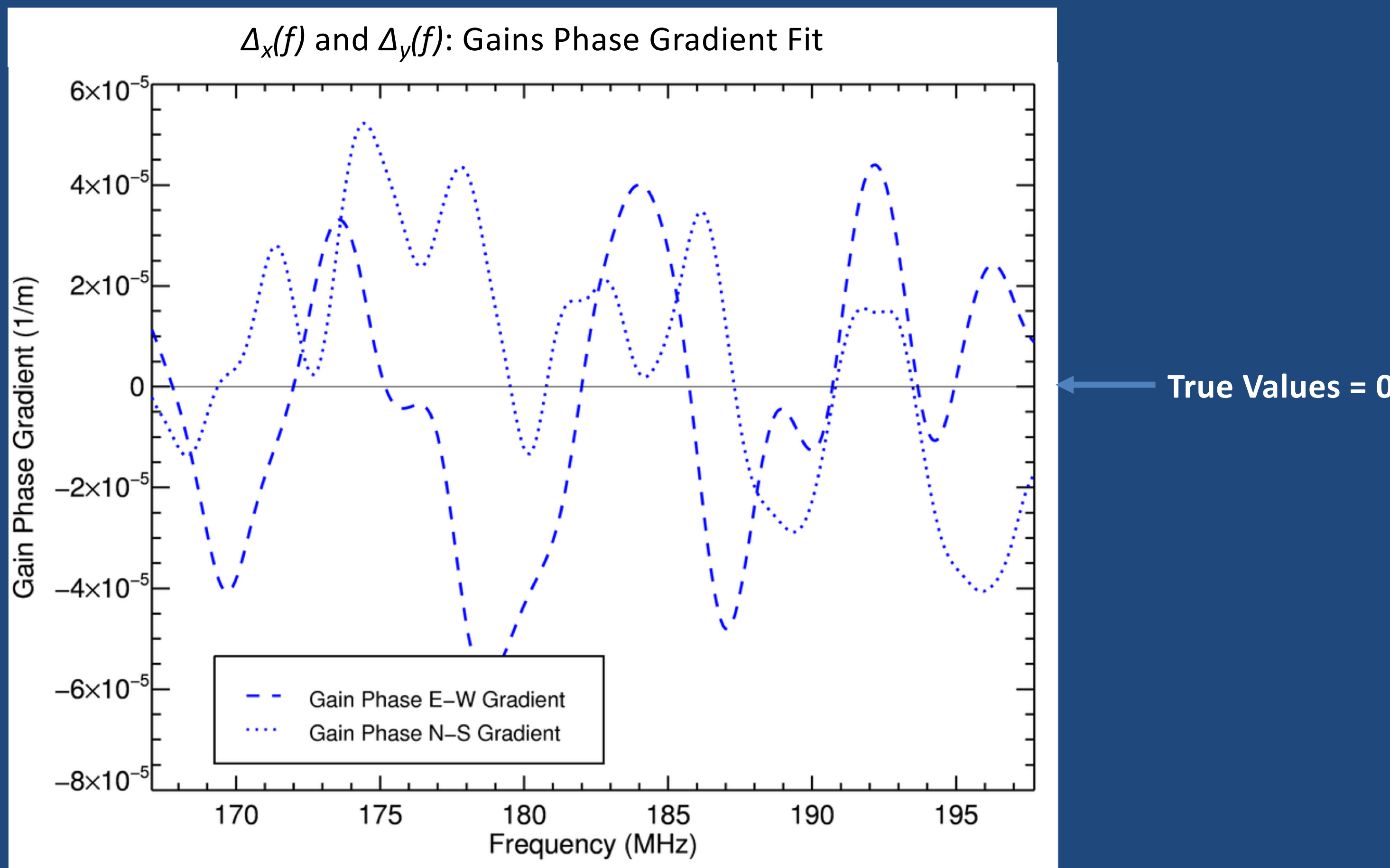
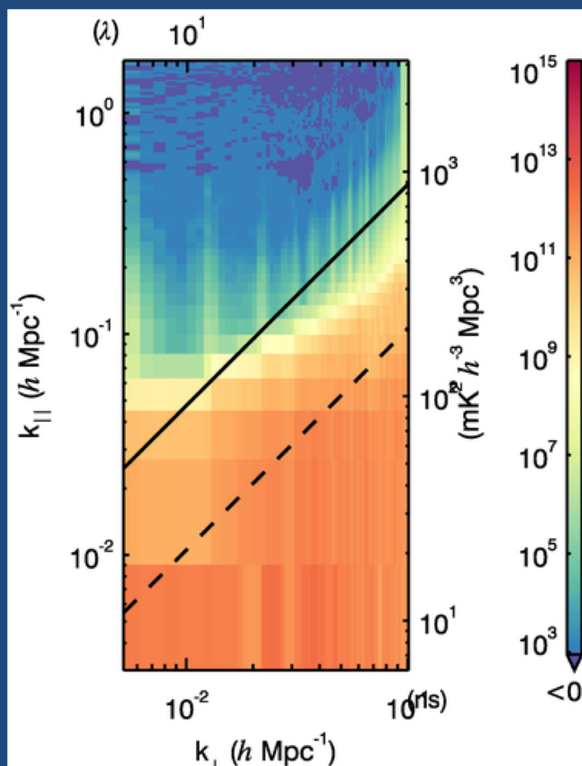
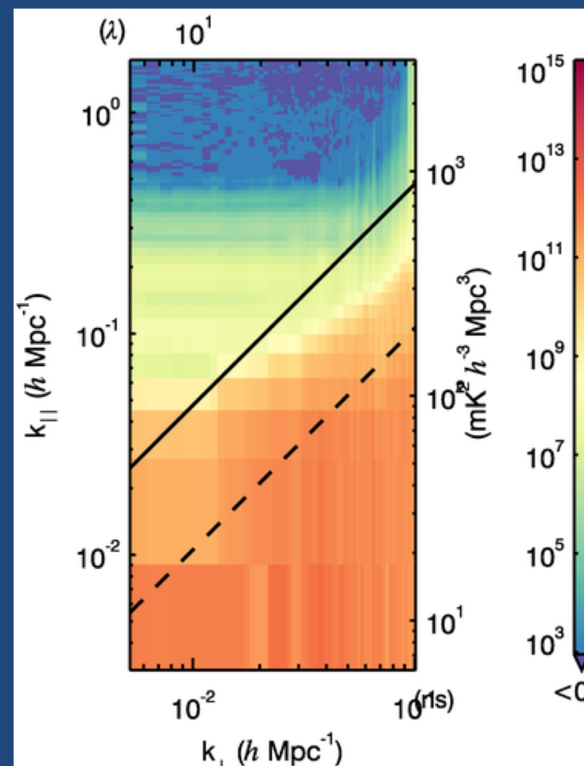


Incomplete Sky Models and Absolute Calibration

Perfect calibration



Errors in absolute calibration parameters
A(f), Δ_x(f), and *Δ_y(f)*



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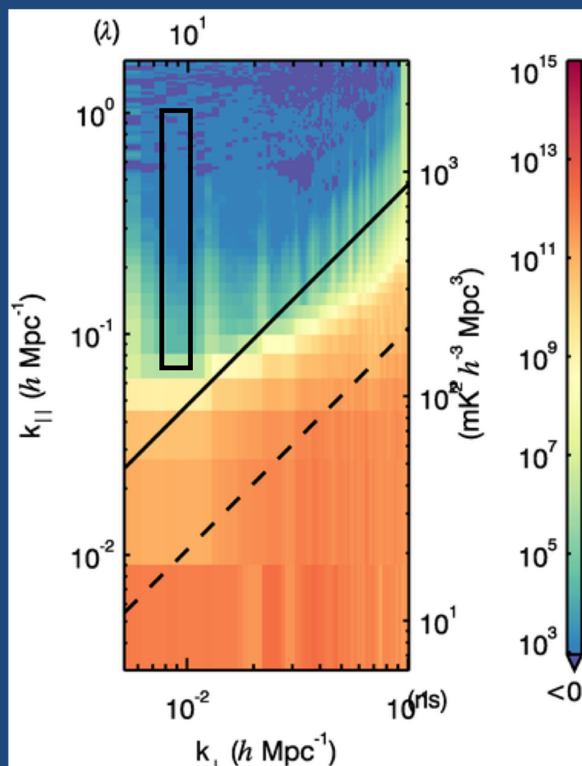
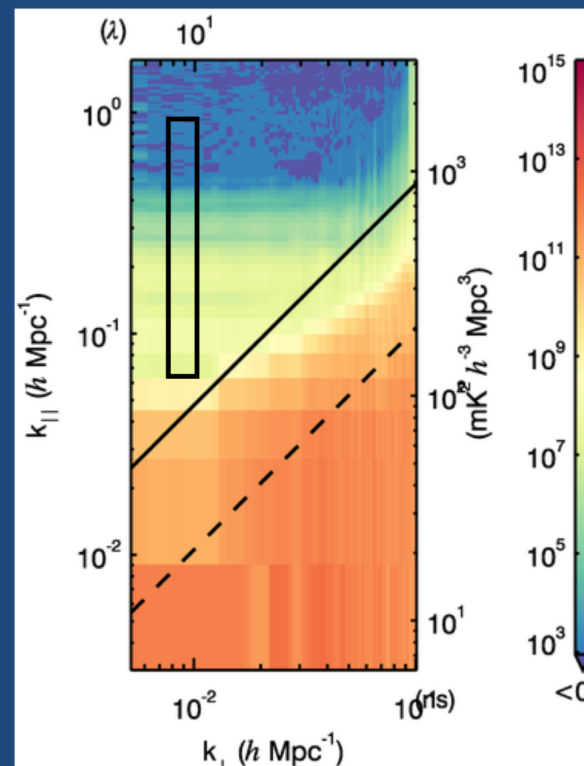


Errors in absolute calibration parameters
A(f), *Δ_x(f)*, and *Δ_y(f)*



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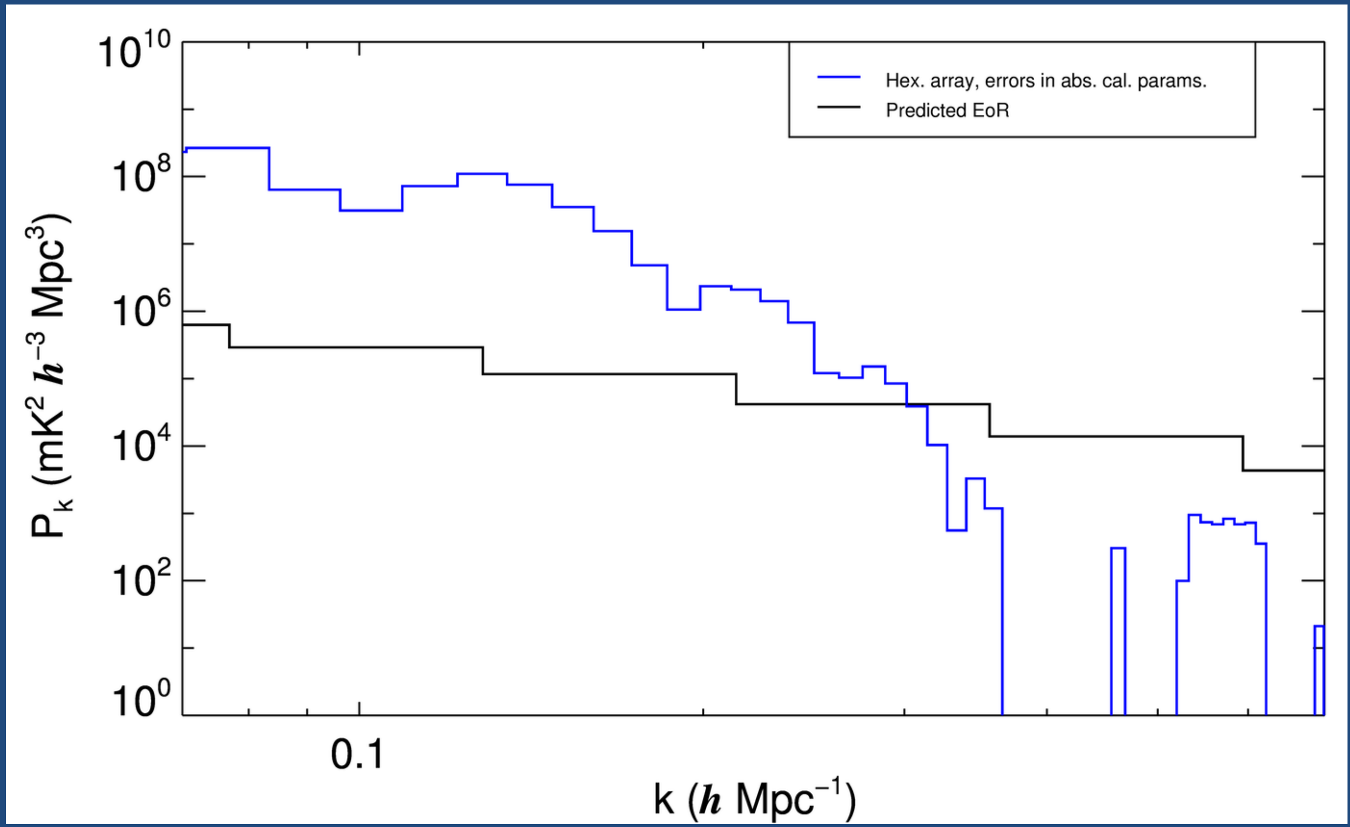
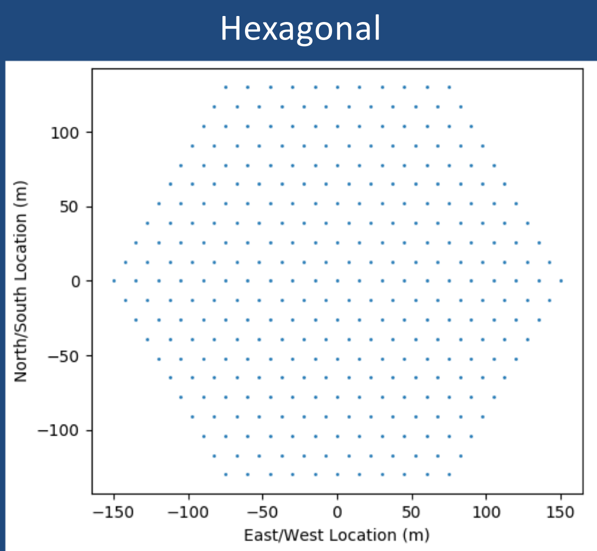
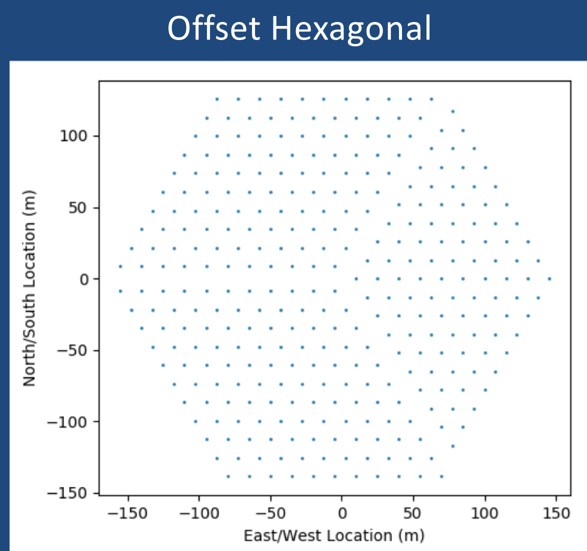
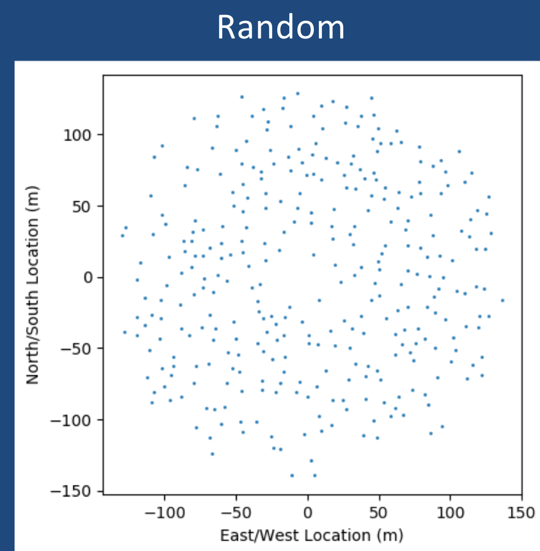


Calibration Errors and Array Layout







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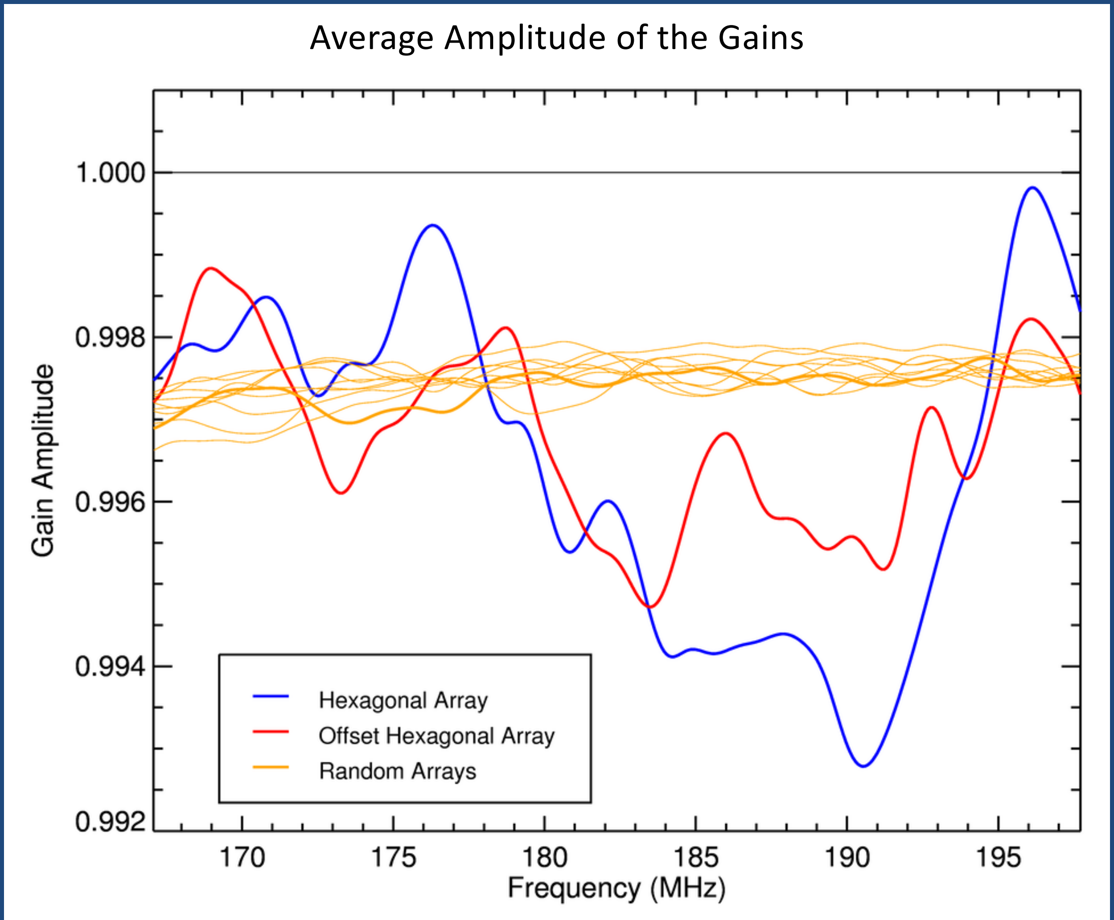
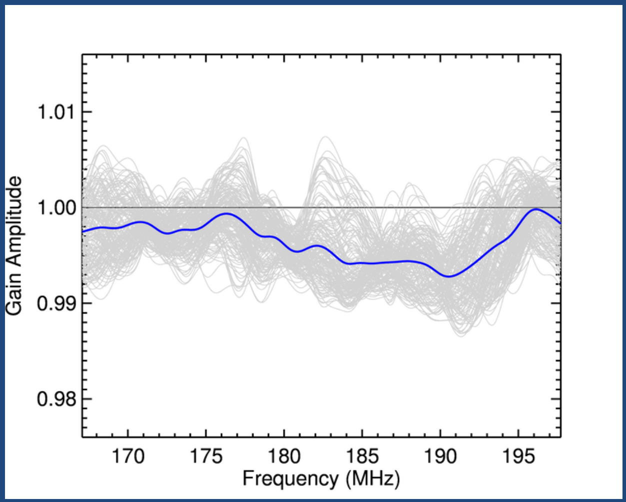


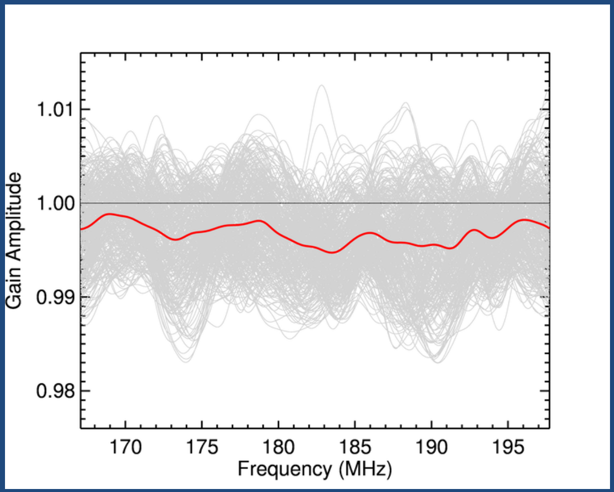
- Random arrays have smaller absolute calibration errors

Calibration Errors and Array Layout

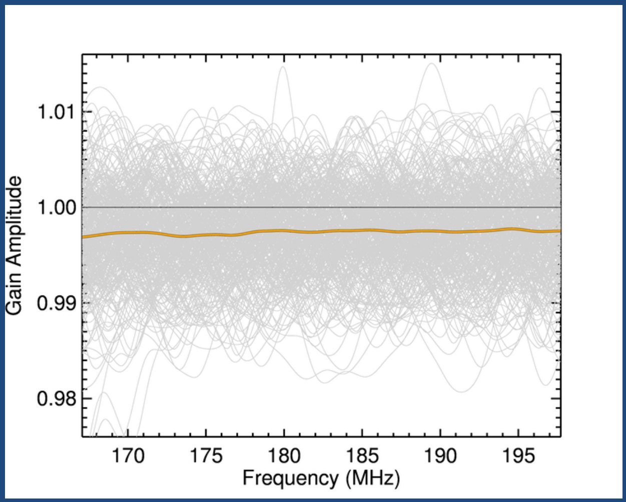
Hexagonal



Offset Hexagonal



Random



- Calibration errors from sky model incompleteness are highly correlated for redundant arrays

What now?

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- **Develop better sky models**

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- Develop better sky models
- **Build arrays with good baseline coverage**

What now?

- Develop better sky models
- Build arrays with good baseline coverage
- **Calibrate only to short baselines**

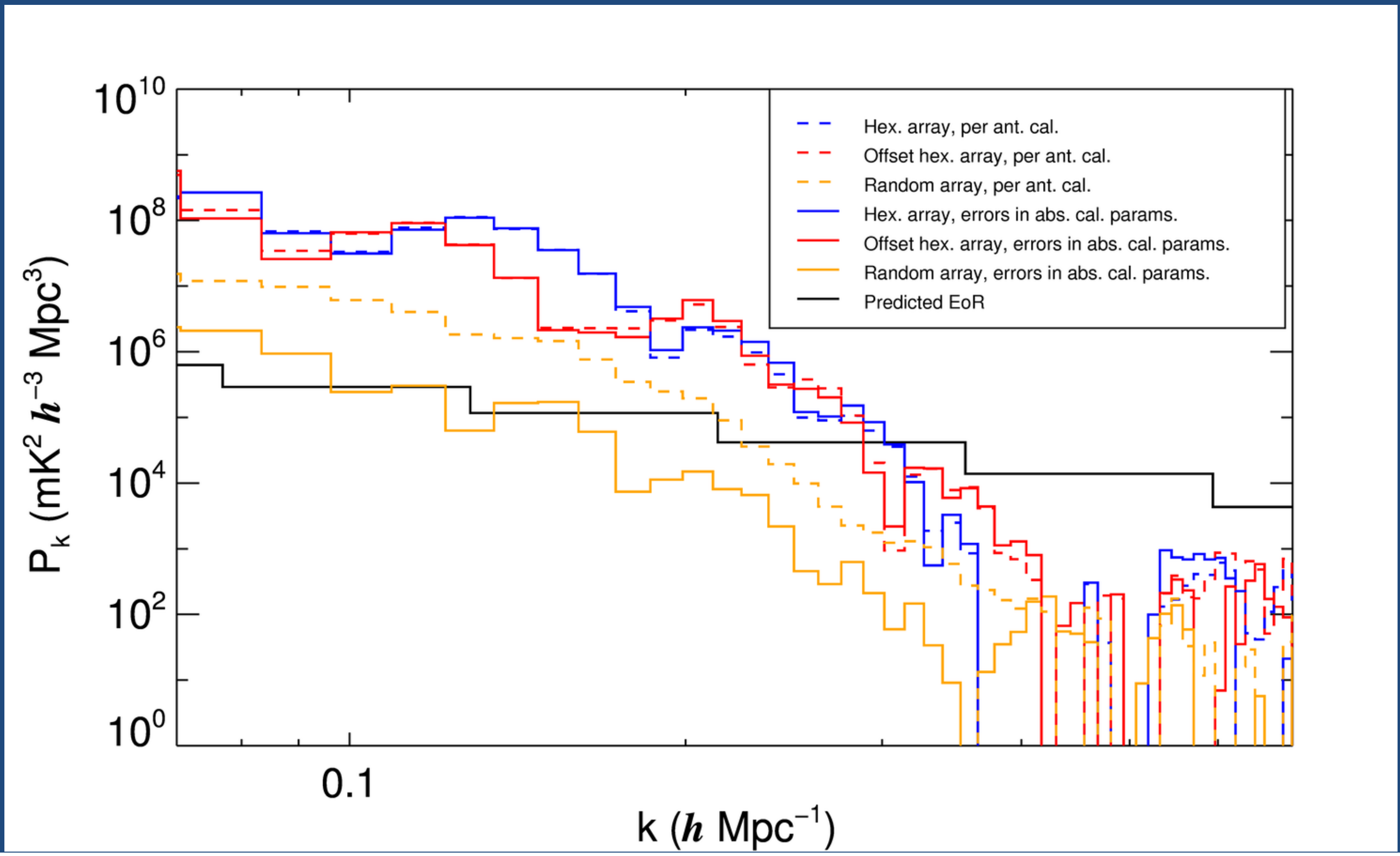
What now?

- Develop better sky models
- Build arrays with good baseline coverage
- Calibrate only to short baselines
- **Build antennas with spectrally smooth responses**

What now?

- Develop better sky models
- Build arrays with good baseline coverage
- Calibrate only to short baselines
- Build antennas with spectrally smooth responses
- **All of the above**

Calibration Errors in Redundant and Per-Antenna Calibration



Absolute Calibration

- Sky-based calibration: $v_{ij}(f) = g_i^{\mathrm{sky}}(f) g_j^{\mathrm{sky}^*}(f) m_{ij}(f) + n_{ij}(f)$
- Average amplitude *A* is the average of the sky-based gain amplitudes:
  $A(f) = \frac{1}{N}\sum_{i=0}^{N} |g_i^{\mathrm{sky}}(f)|$
- Average phase *Δ* is degenerate for both sky-based and redundant calibration
  - *Use a reference antenna*
  - *Use Δ=0*
- Phase gradient parameters are fit to the phases of the sky-based gains: $\chi^2_\phi(f) = \sum_{i=0}^{N}\left(\mathrm{Arg}[g_i^{\mathrm{sky}}(f)] - \Delta(f) - \Delta_x(f)x_i - \Delta_y(f)y_i\right)^2$