Precision Calibration for 21 cm Cosmology

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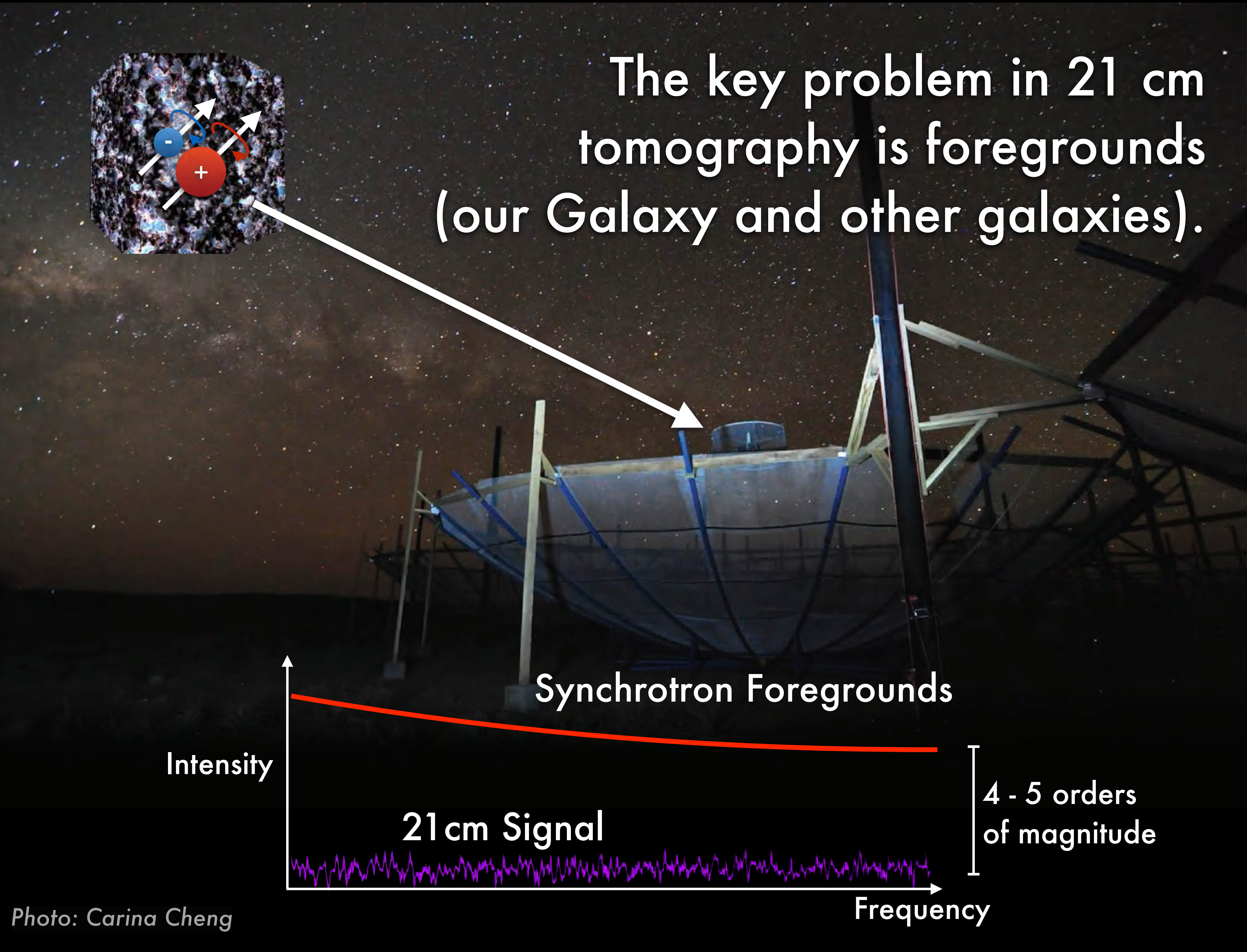
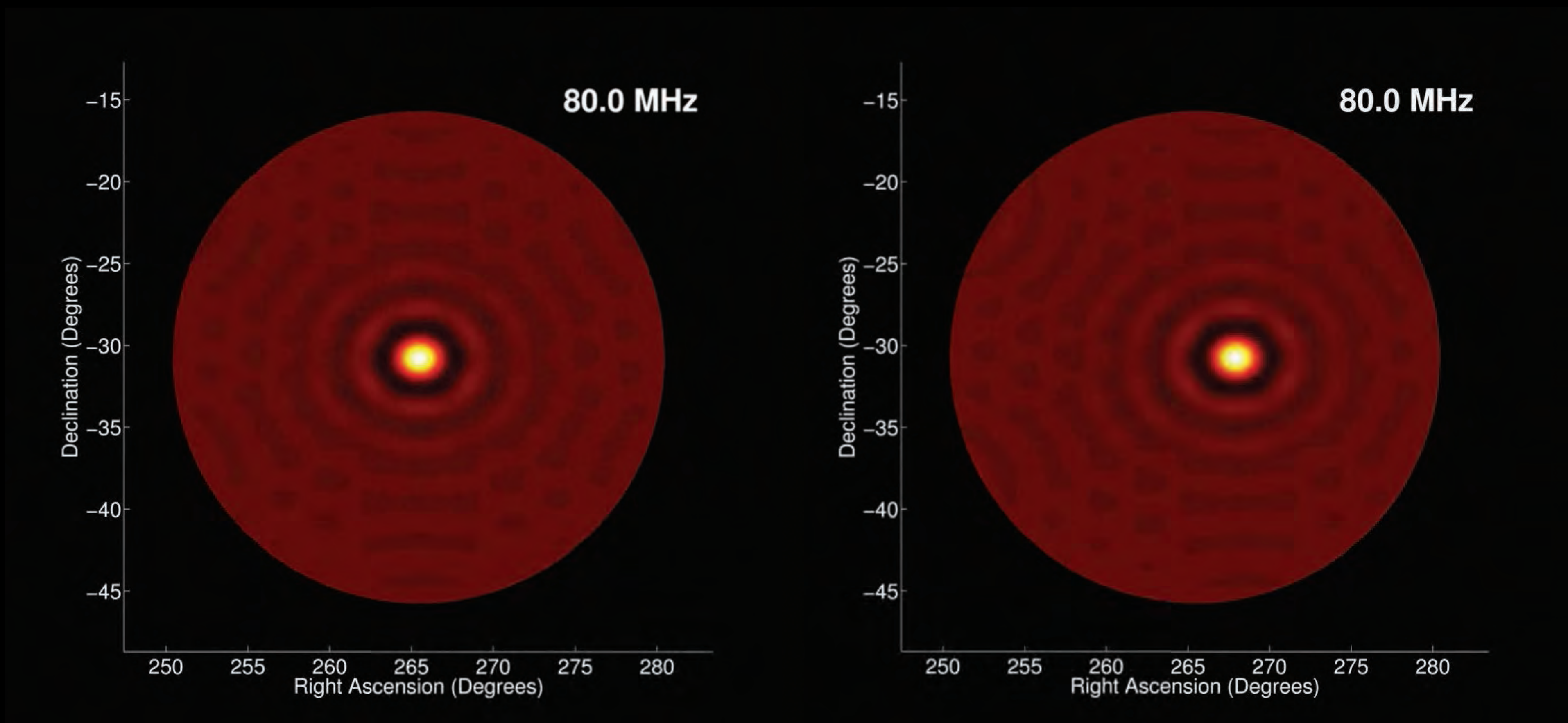
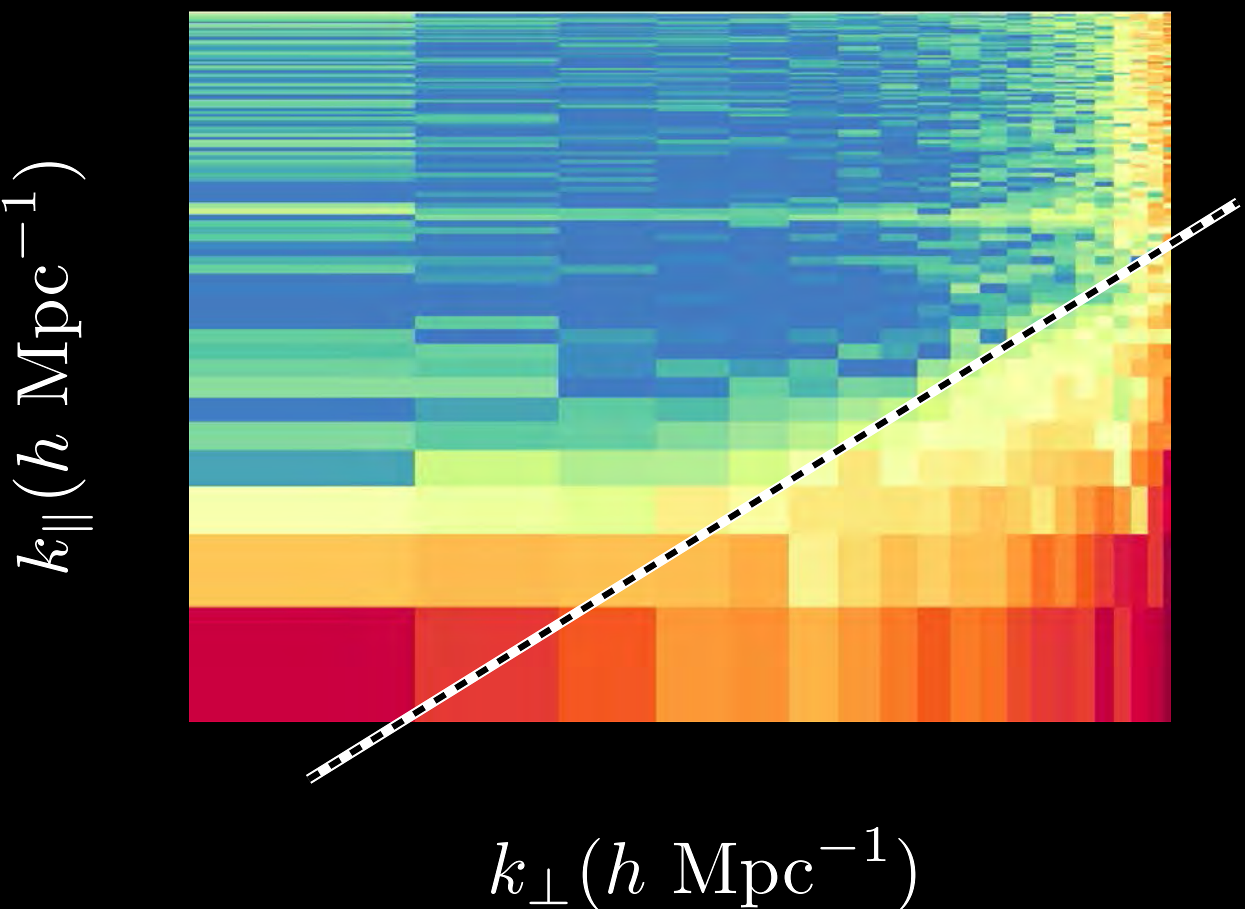

The key problem in 21 cm tomography is foregrounds (our Galaxy and other galaxies).
-
+
Synchrotron Foregrounds
Intensity
4 - 5 orders of magnitude
21cm Signal
Frequency
Photo: Carina Cheng

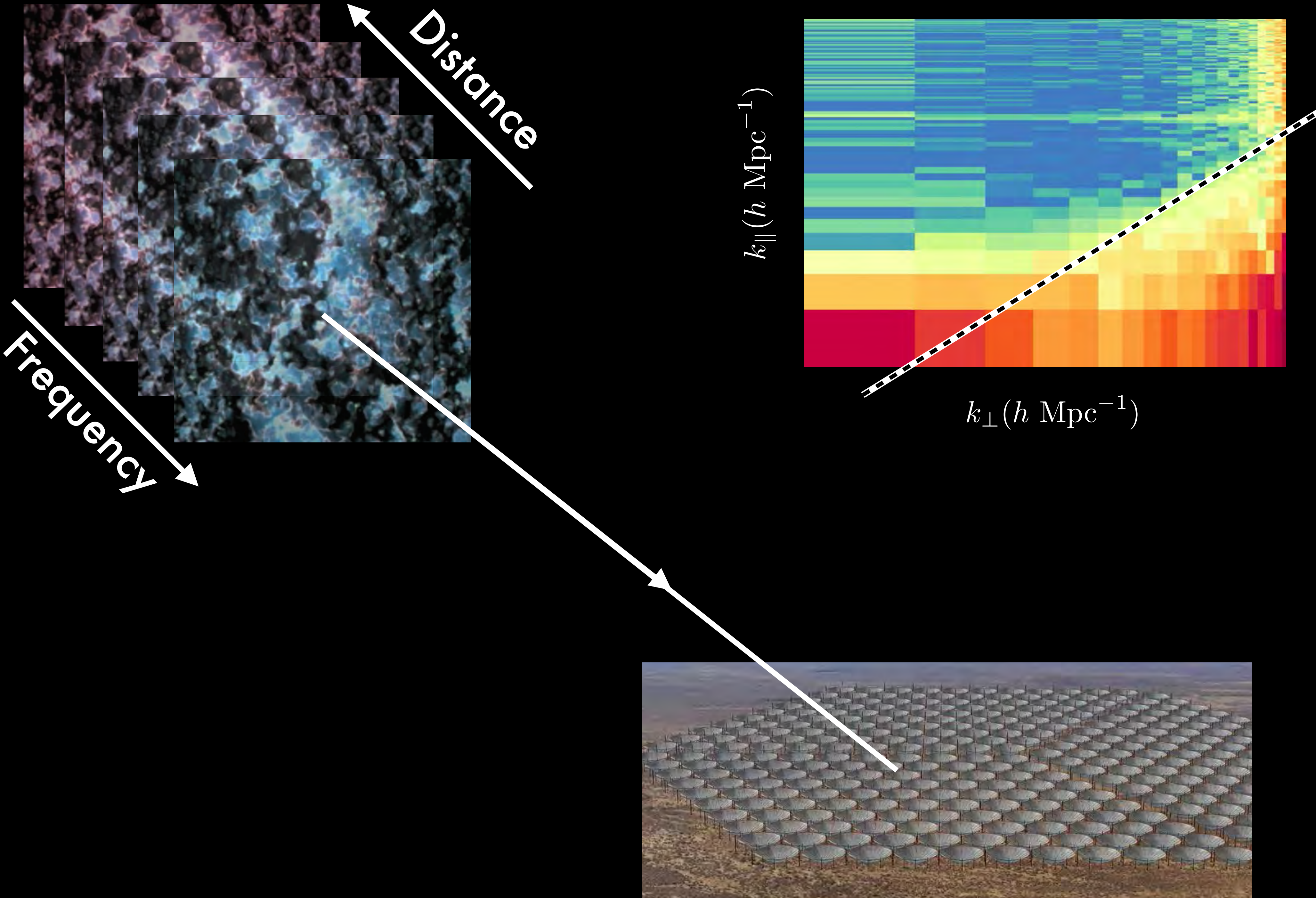
However, a frequency-dependent PSF creates spectral structure in smooth foregrounds.

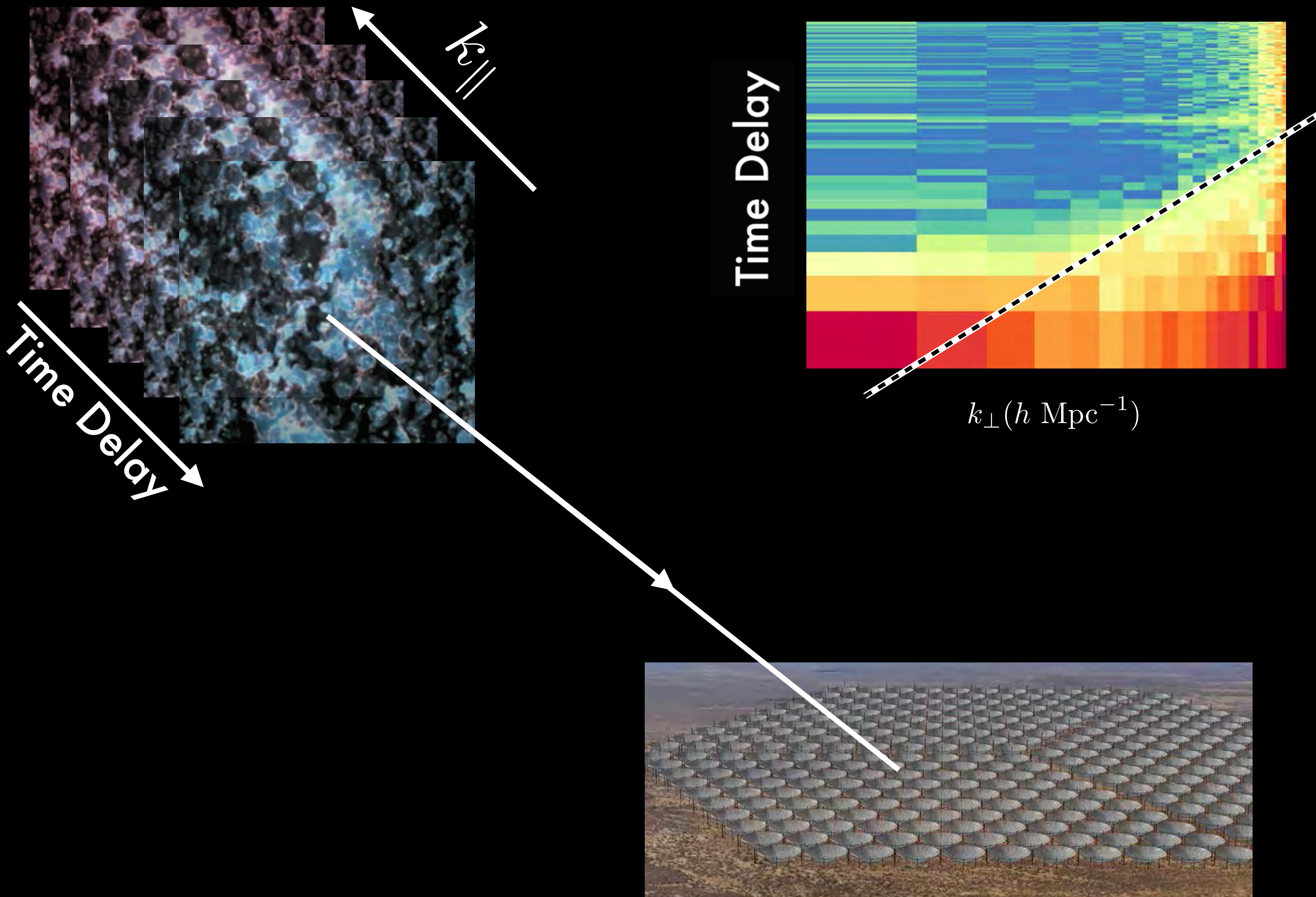


Dillon et al. (2015)

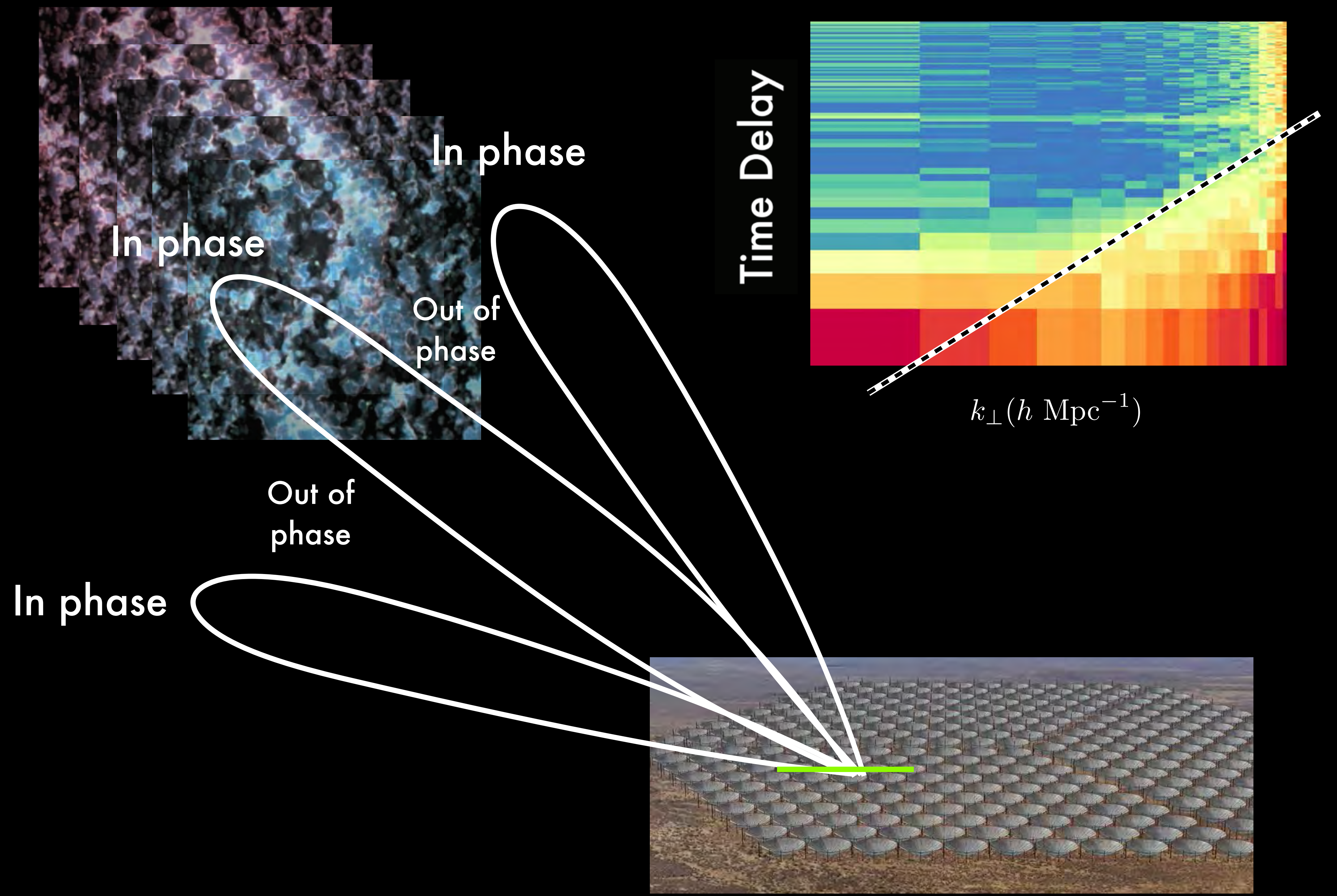
This creates the foreground *wedge* in the 2-D power spectrum.

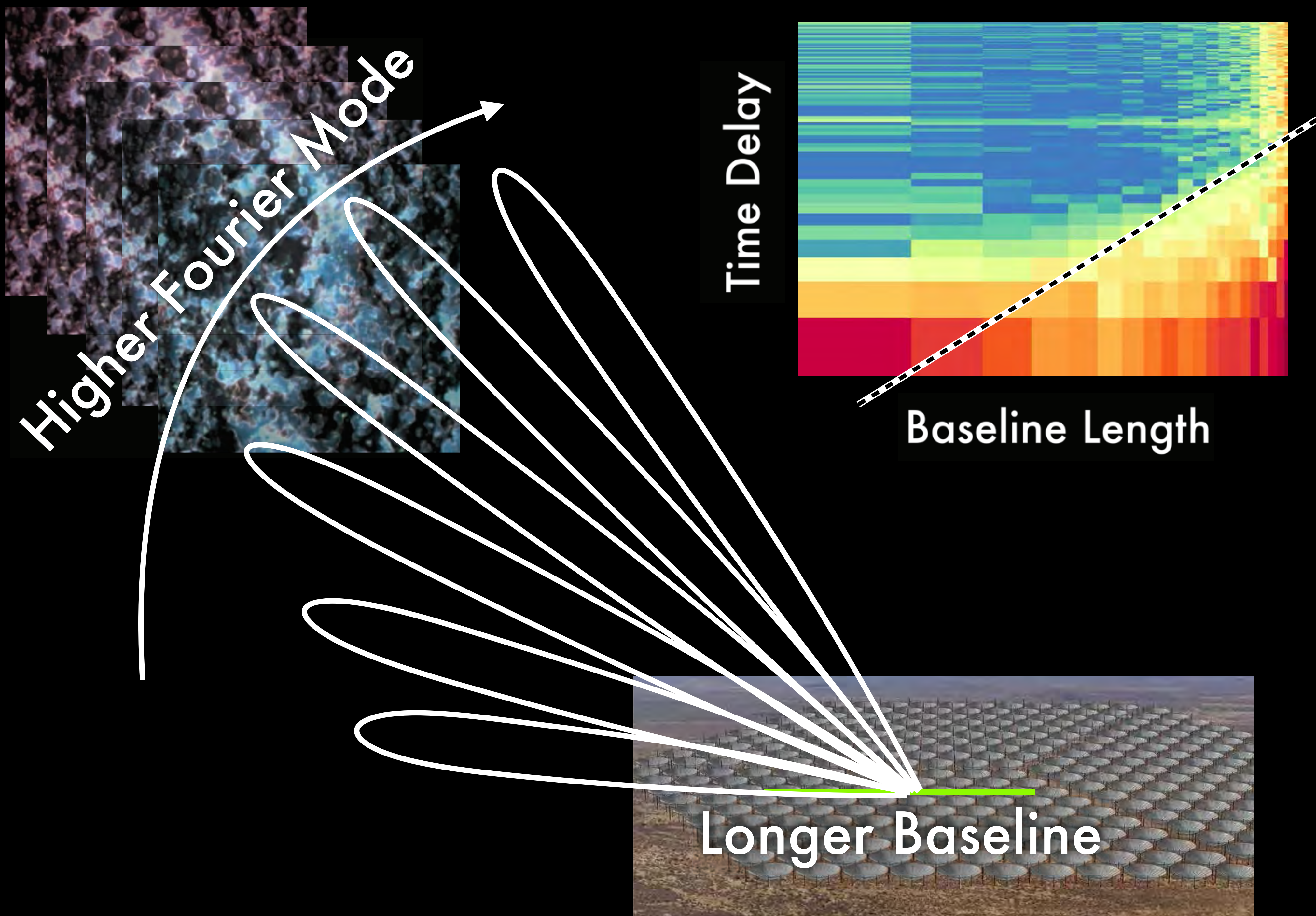


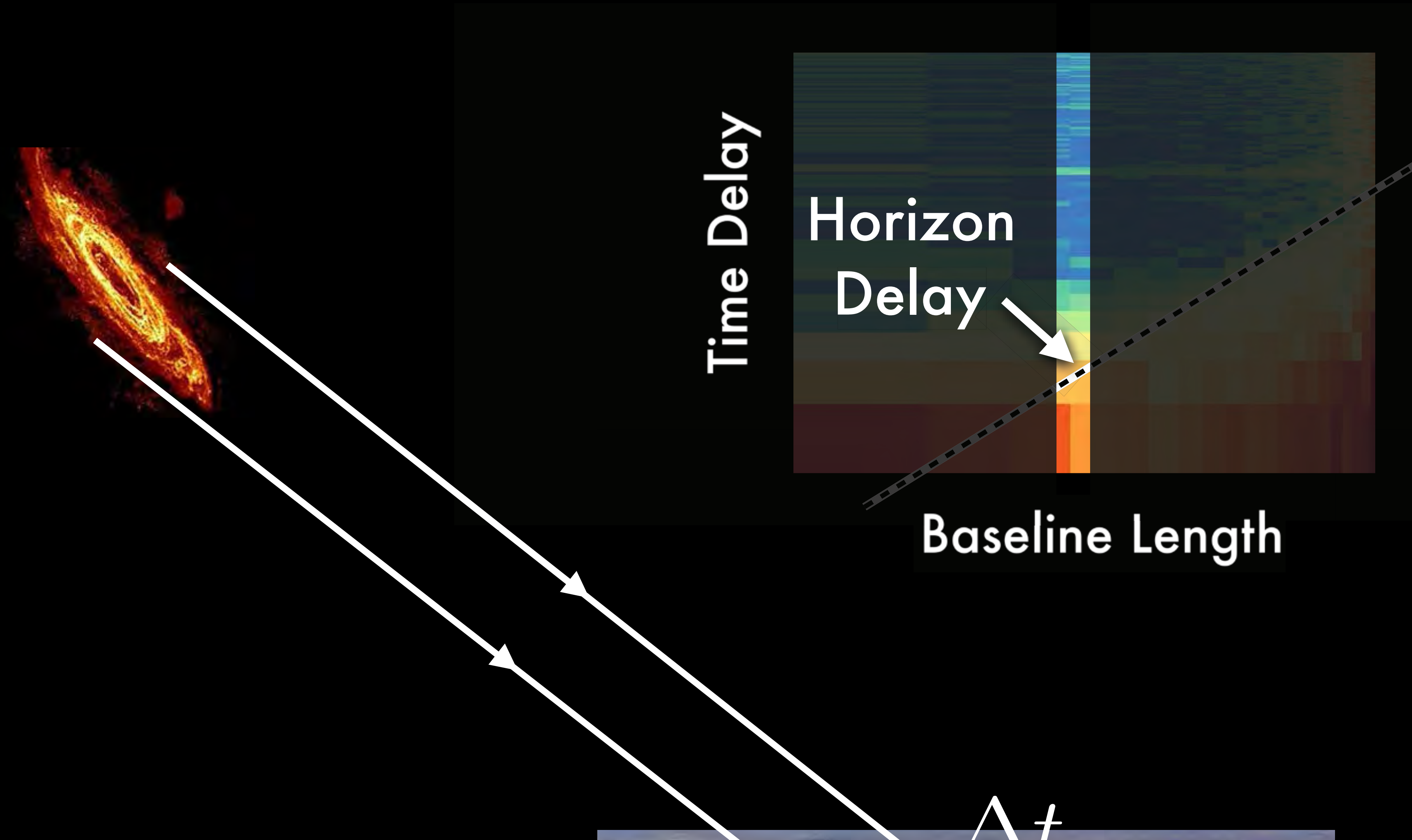

Distance
Frequency
$k_{\parallel}(h\ \mathrm{Mpc}^{-1})$
$k_{\perp}(h\ \mathrm{Mpc}^{-1})$

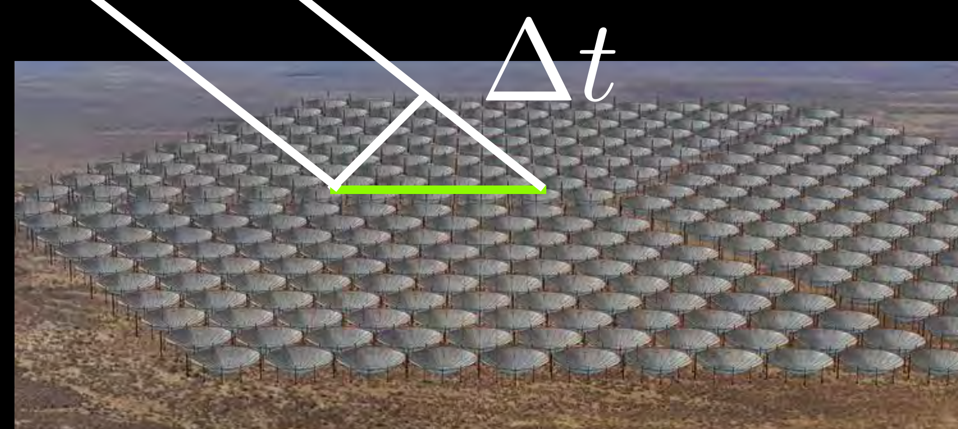


$k_{\parallel}$
Time Delay
Time Delay
$k_{\perp} (h\ \mathrm{Mpc}^{-1})$


In phase
In phase
Out of phase
Out of phase
In phase
Time Delay
$k_\perp (h\ \mathrm{Mpc}^{-1})$

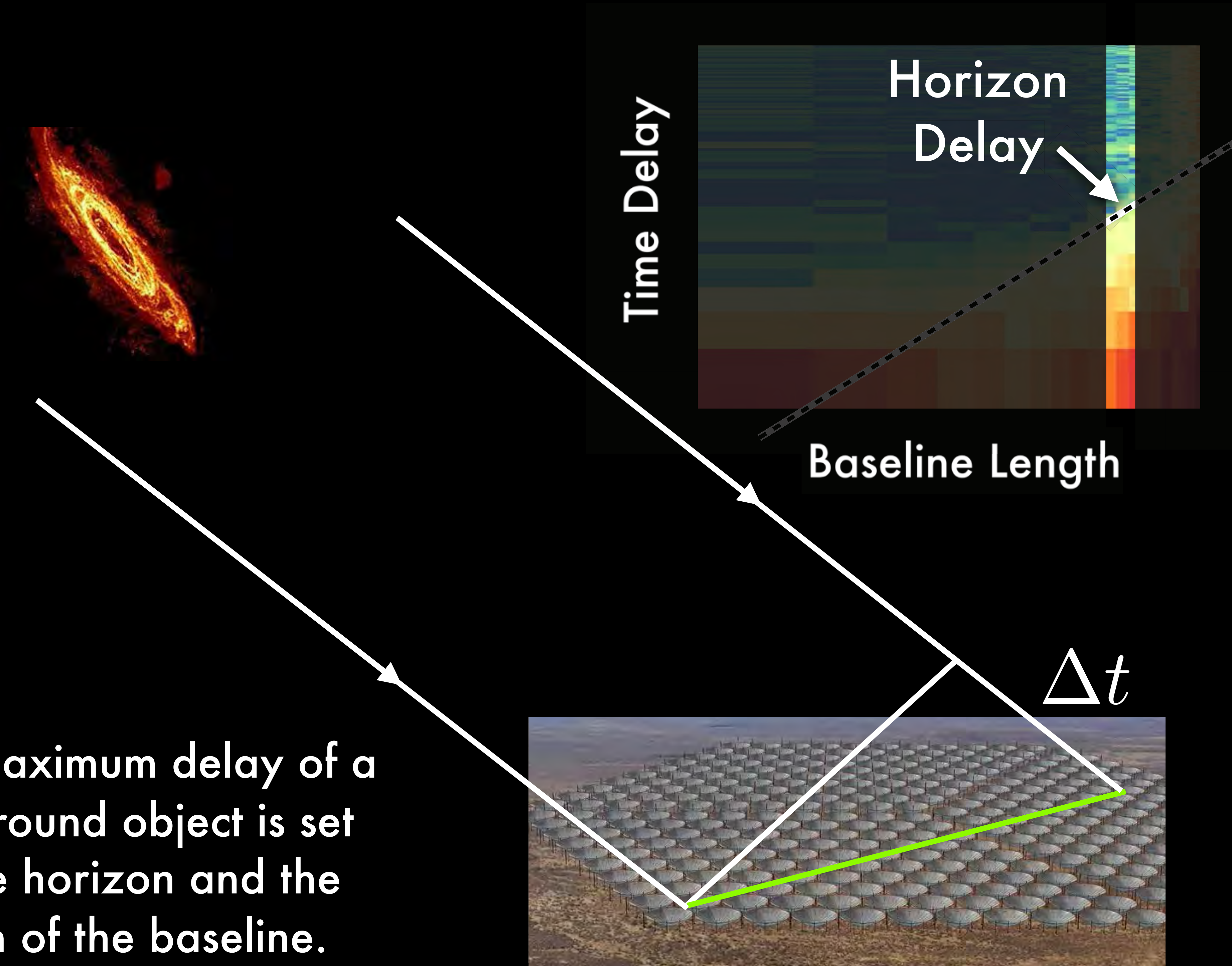

Higher Fourier Mode
Time Delay
Baseline Length
Longer Baseline





The maximum delay of a foreground object is set by the horizon and the length of the baseline.

Parsons et al. (2012)



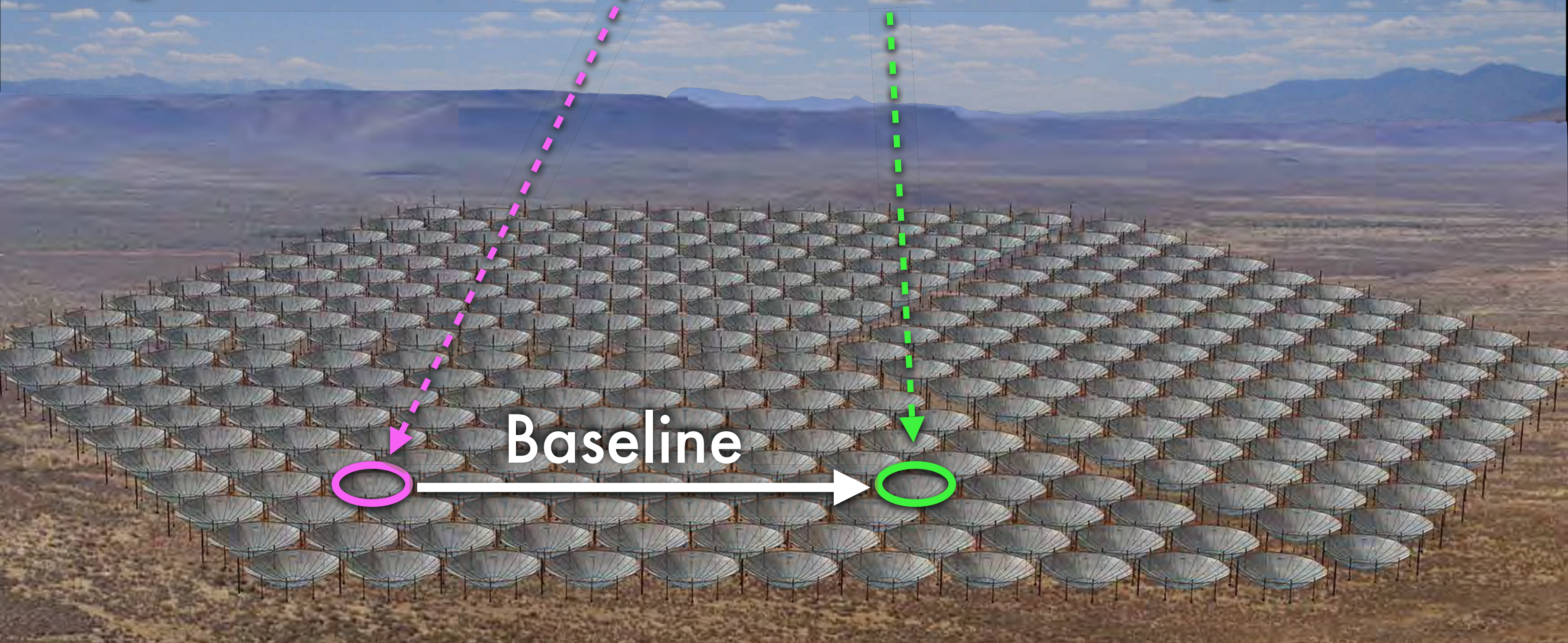
The maximum delay of a foreground object is set by the horizon and the length of the baseline.

Parsons et al. (2012)

The key to foreground mitigation
is a smooth instrumental response.

Calibration is key to spectral smoothness.

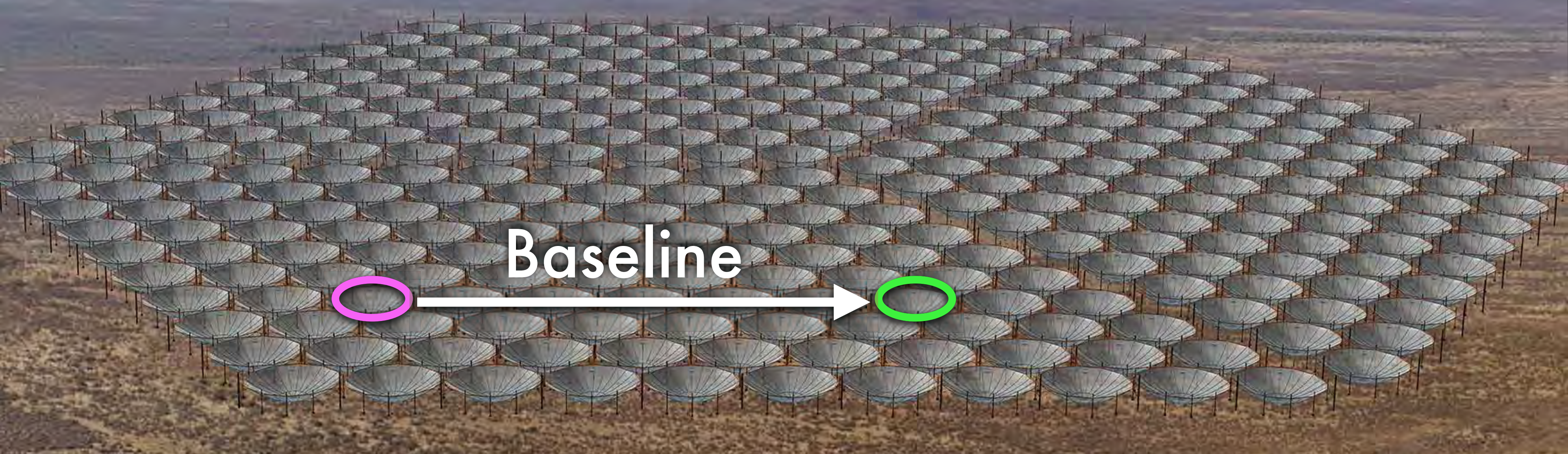
$$V_{ij}^{\mathrm{obs}}(\nu) = g_i(\nu) g_j^*(\nu) V_{ij}^{\mathrm{true}}(\nu)$$



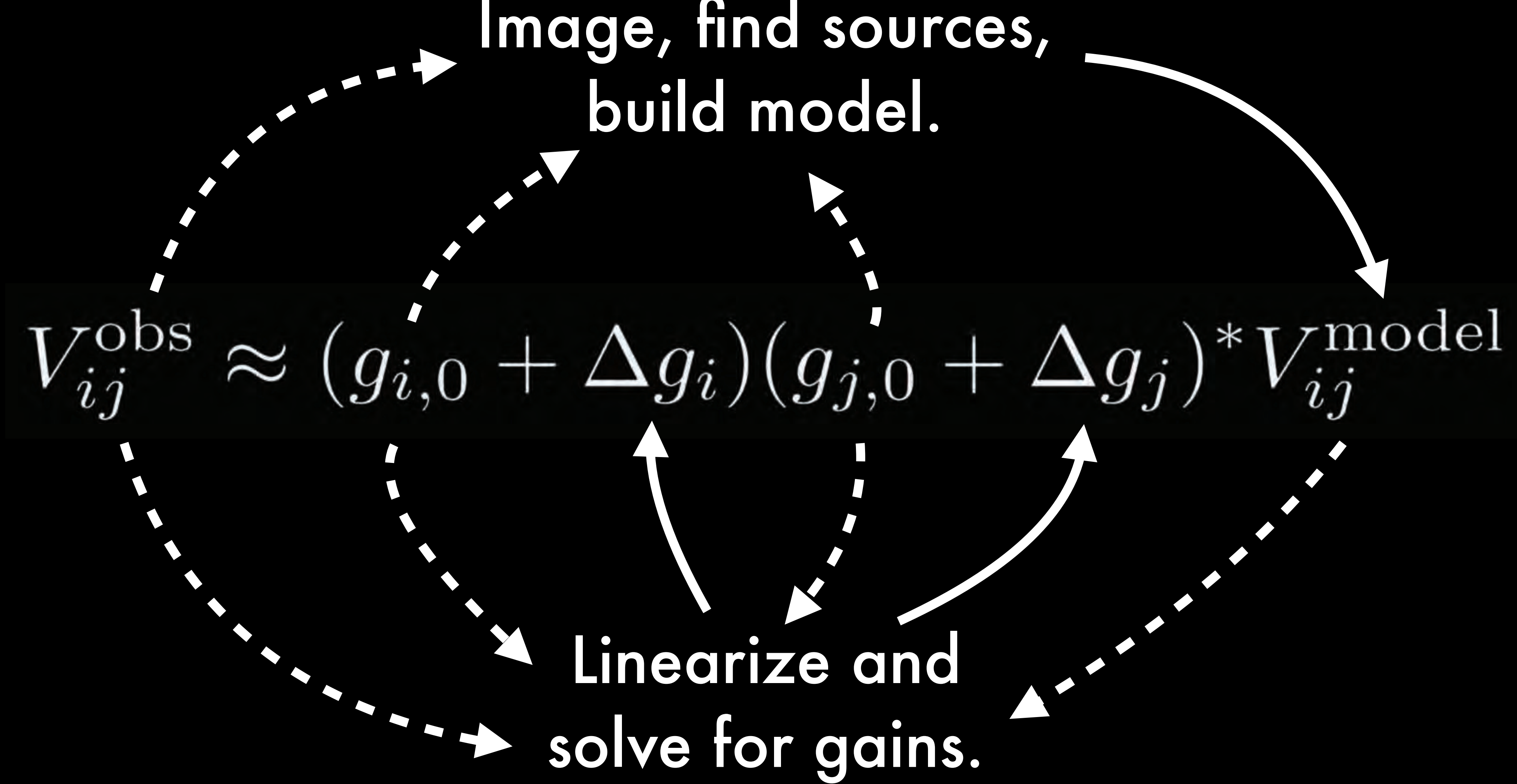
Calibration is key to spectral smoothness.

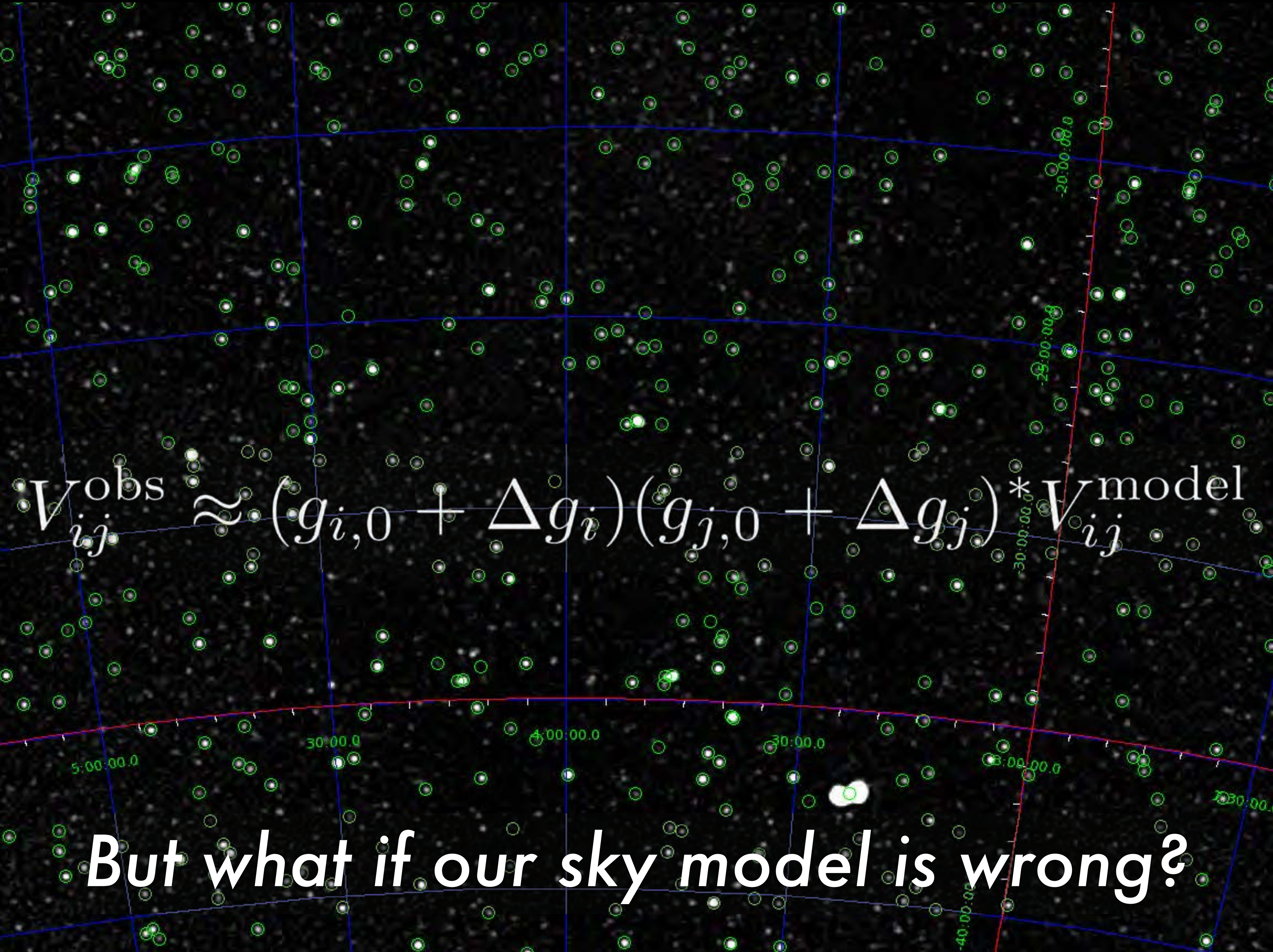
$$V_{ij}^{\text{obs}}(\nu) = g_i(\nu) g_j^*(\nu) V_{ij}^{\text{true}}(\nu)$$

$$V_{ij}^{\text{obs}} \approx (g_{i,0} + \Delta g_i)(g_{j,0} + \Delta g_j)^* V_{ij}^{\text{model}}$$



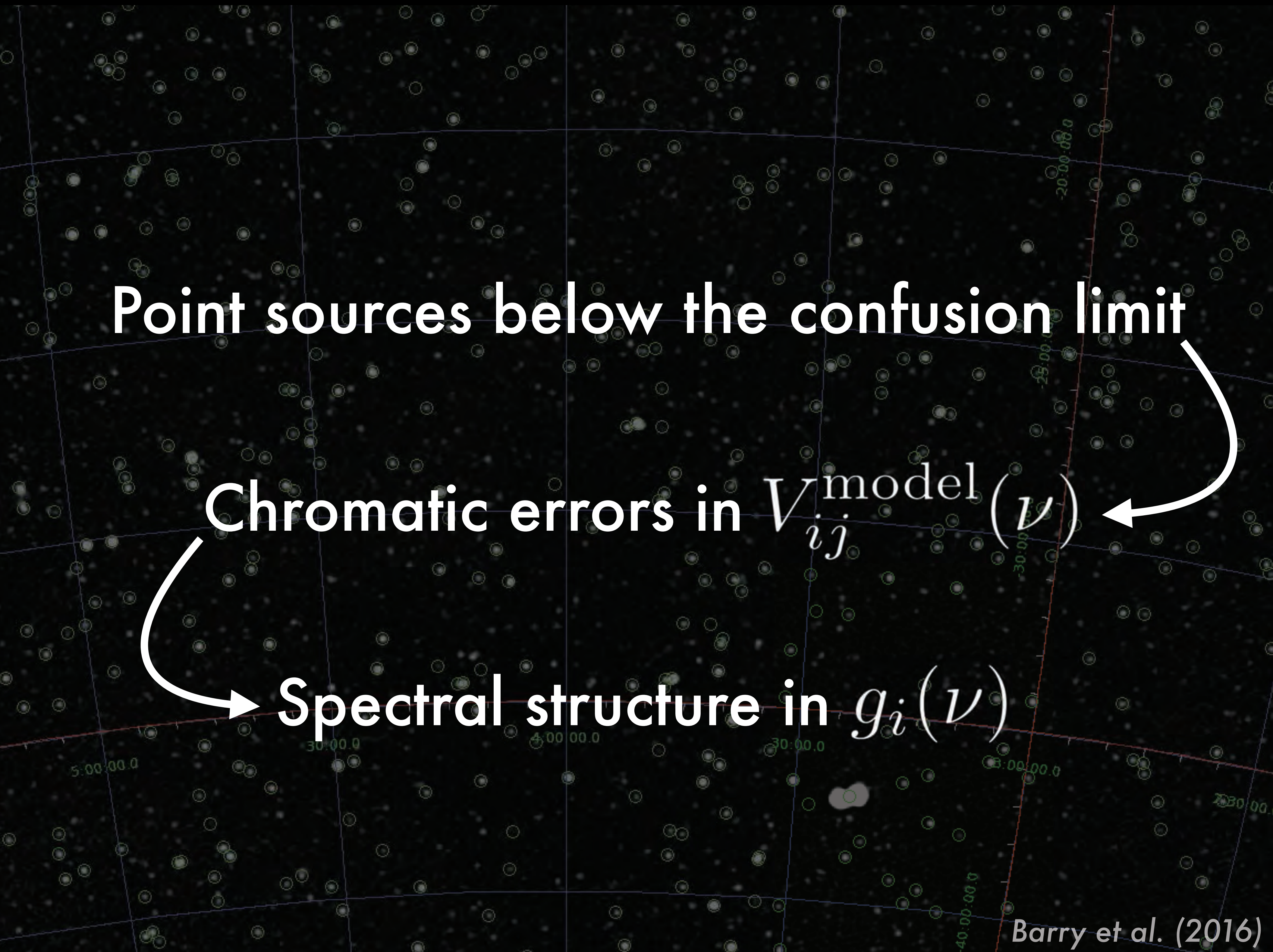
The Self-Cal Loop





$$V_{ij}^{\mathrm{obs}} \approx (g_{i,0} + \Delta g_i)(g_{j,0} + \Delta g_j)^* V_{ij}^{\mathrm{model}}$$

But what if our sky model is wrong?



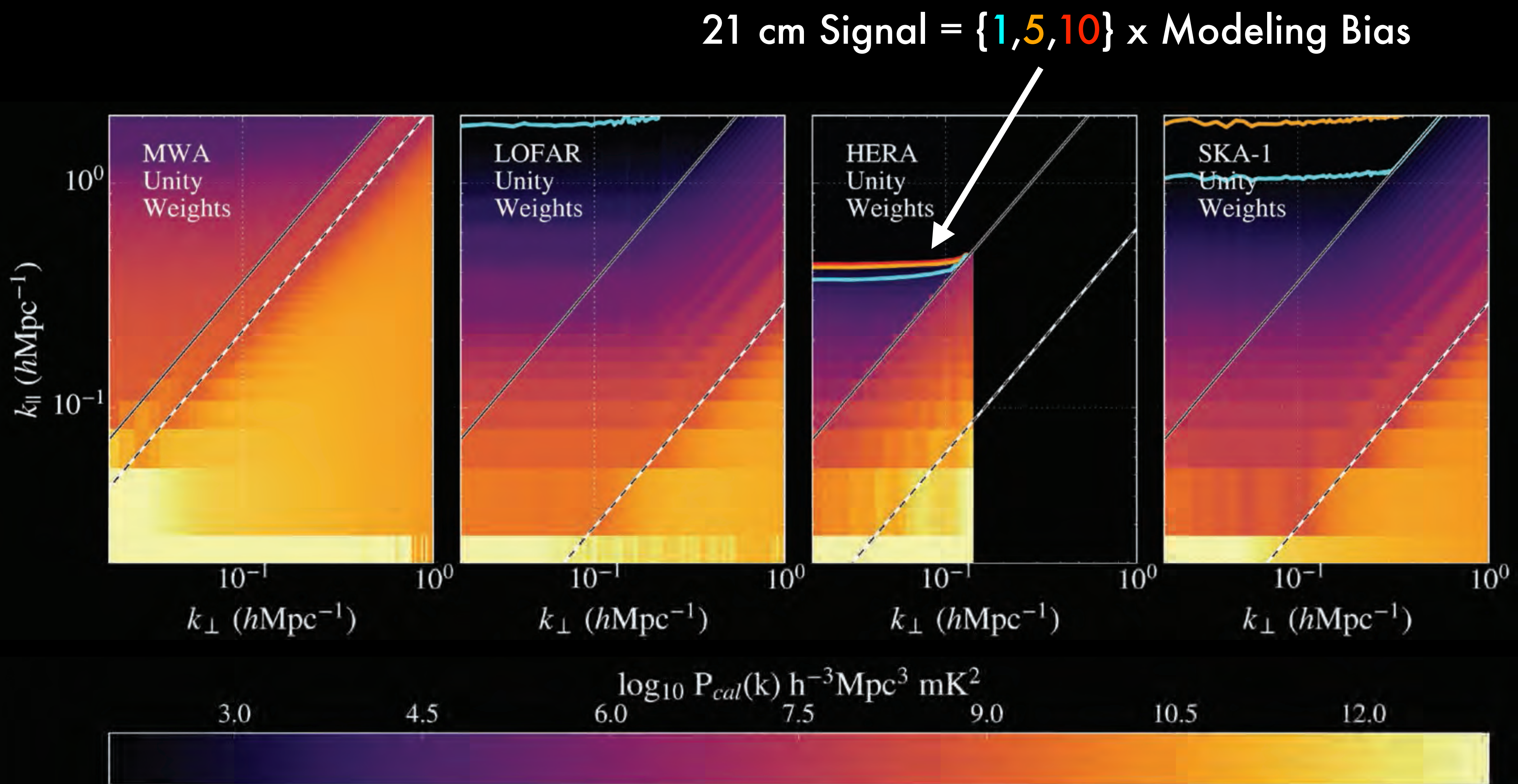
Point sources below the confusion limit

Chromatic errors in $V_{ij}^{\mathrm{model}}(\nu)$

Spectral structure in $g_i(\nu)$

Barry et al. (2016)

Structure in $g_i(\nu)$ given by longest baseline b_{ij}.
Modeling error turns the wedge into a brick.



Ewall-Wice, Dillon, Liu, Hewitt (2016)

When linearizing and minimizing χ^2...

$$V_{ij}^{\mathrm{obs}} \approx (g_{i,0} + \Delta g_i)(g_{j,0} + \Delta g_j)^* V_{ij}^{\mathrm{model}}$$

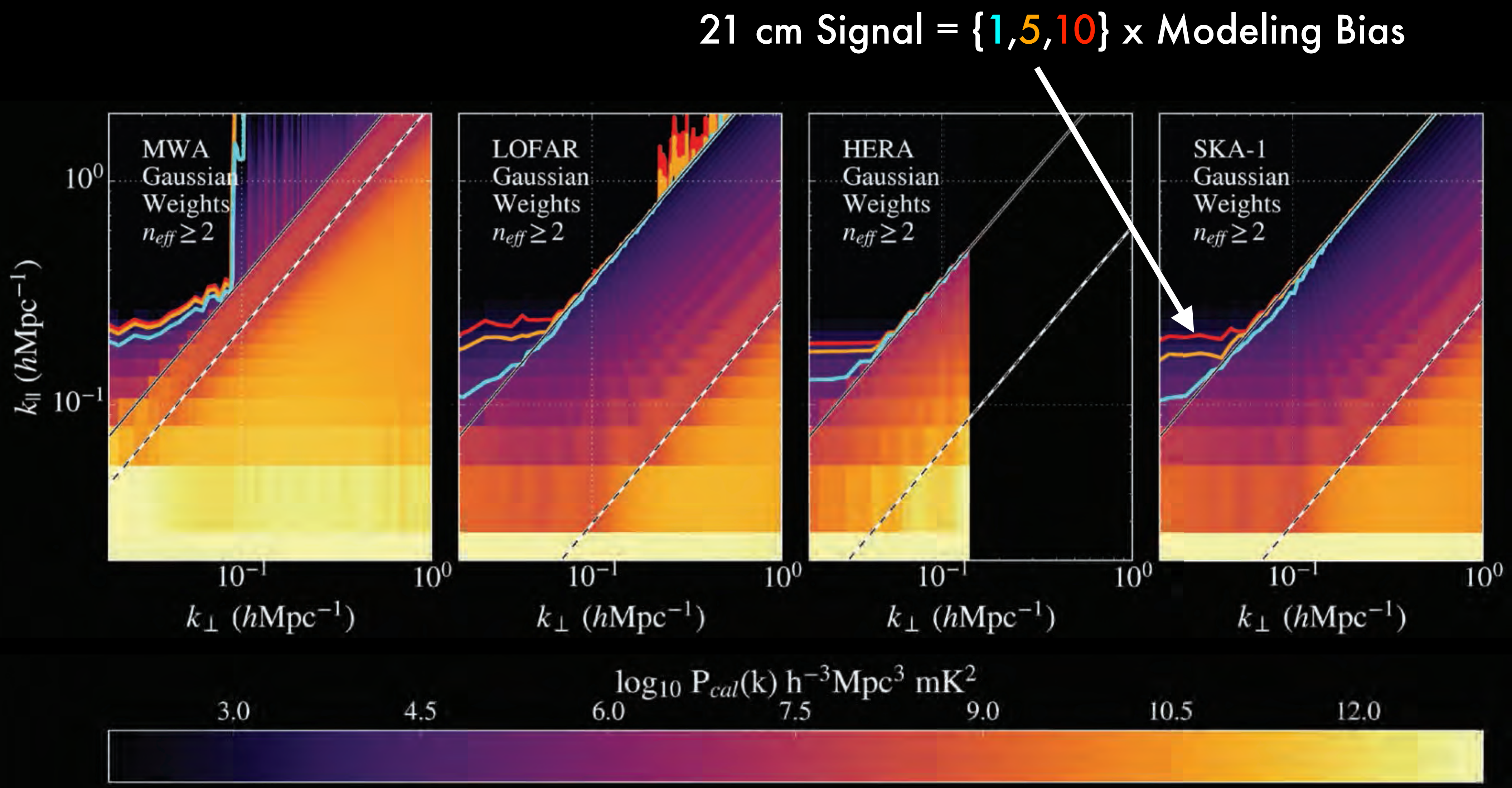
weight each equation in the system by

$$W_{ij} \propto e^{-b_{ij}^2/2\sigma_b^2}$$

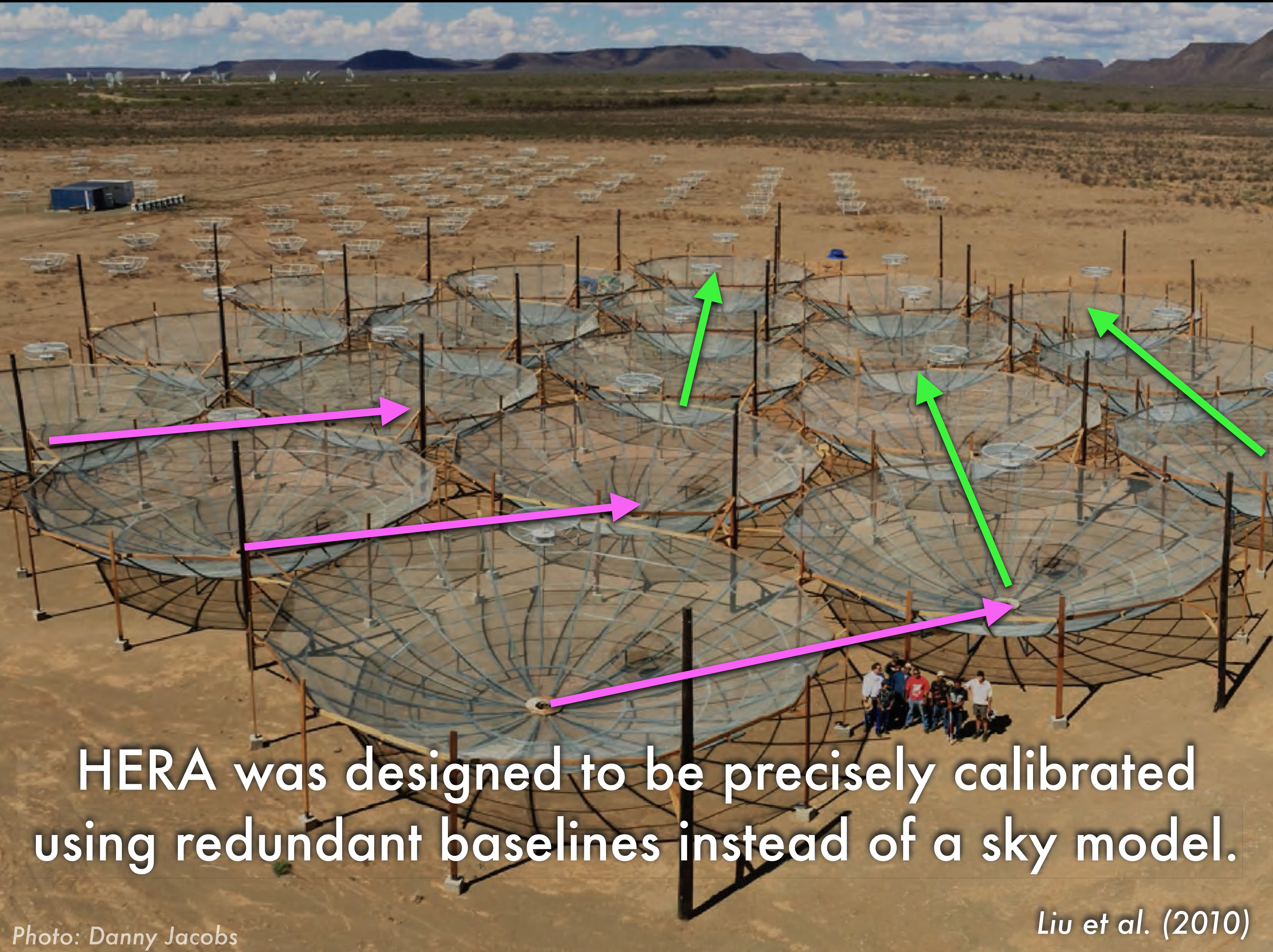
to suppress gain chromaticity leakage from long to short baselines.

Ewall-Wice, Dillon, Liu, Hewitt (2016)

We can recover most of the EoR window for only a modest increase in noise.



Ewall-Wice, Dillon, Liu, Hewitt (2016)


HERA was designed to be precisely calibrated
using redundant baselines instead of a sky model.
Liu et al. (2010)
Photo: Danny Jacobs

Many Observations

Few Unique Baselines

$$V_{ij}^{\mathrm{obs}}(\nu) = g_i(\nu) g_j^*(\nu) V_{ij}^{\mathrm{true}}(\nu) + n_{ij}$$

Linearize...

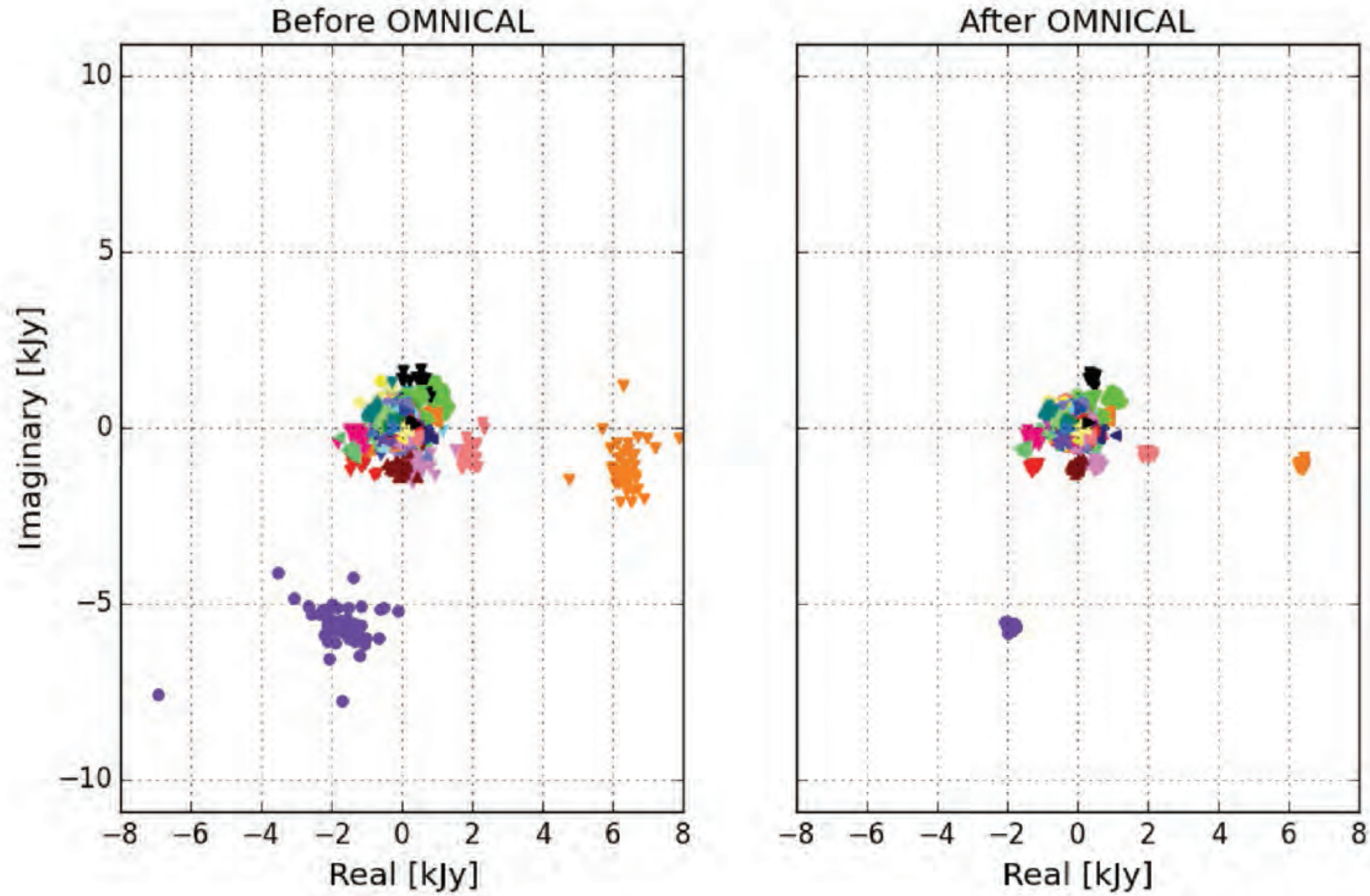
$$\log\left[V_{ij}^{\mathrm{obs}}(\nu)\right] = \log\left[g_i(\nu)\right] + \log\left[g_j^*(\nu)\right] + \log\left[V_{ij}^{\mathrm{model}}(\nu)\right] + n'_{ij}$$

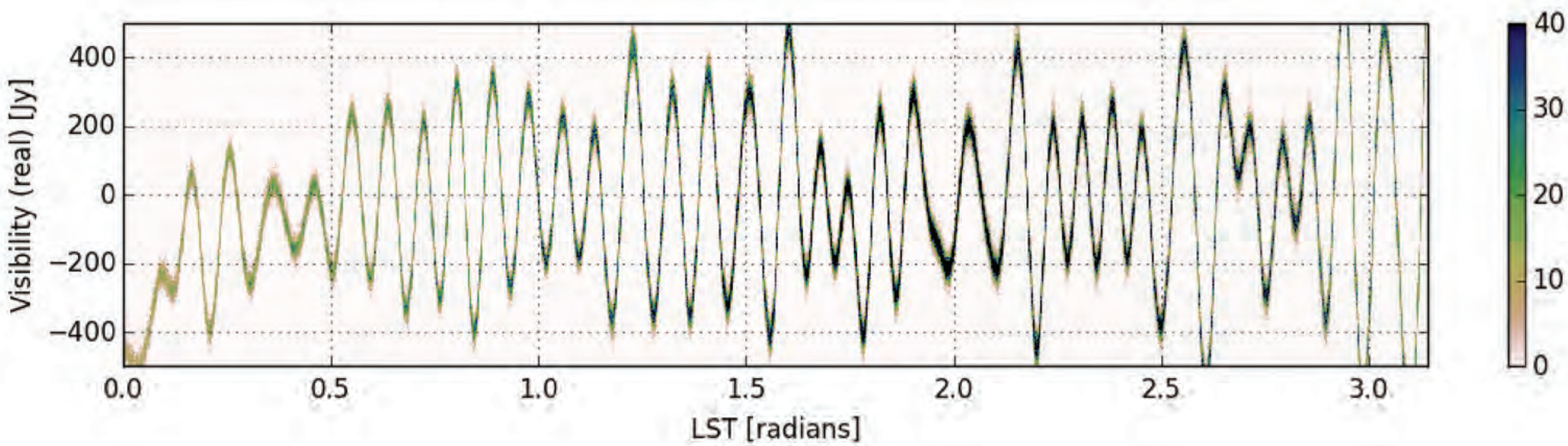
...and then solve for both gains and unique visibilities simultaneously

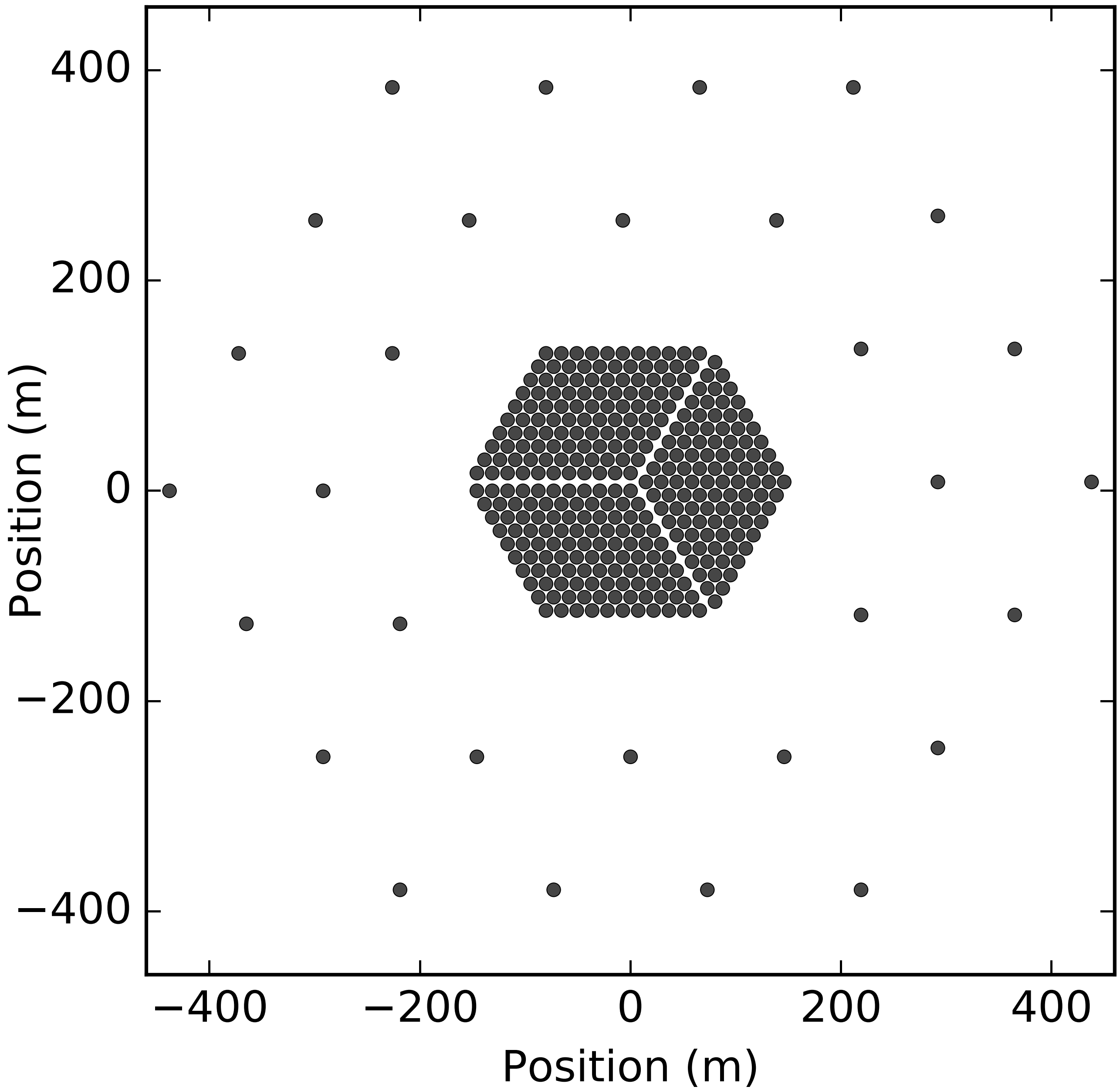
Liu et al. (2010)

Photo: Danny Jacobs

Redundant calibration was key to PAPER-64's power spectrum limits in Ali et al. (2015)





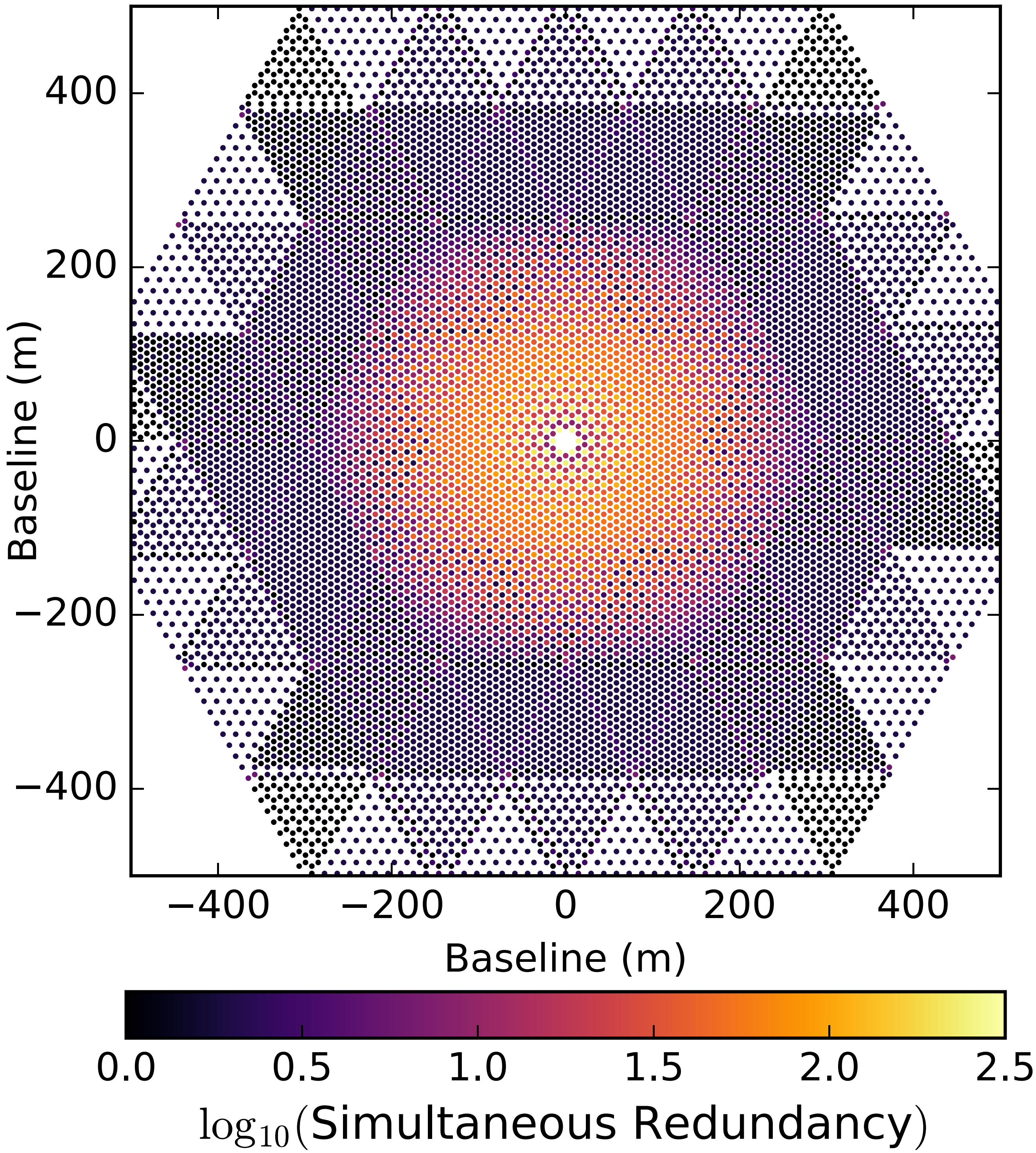


HERA's split configuration is designed for dense sampling of the uv-plane and redundant calibratability.

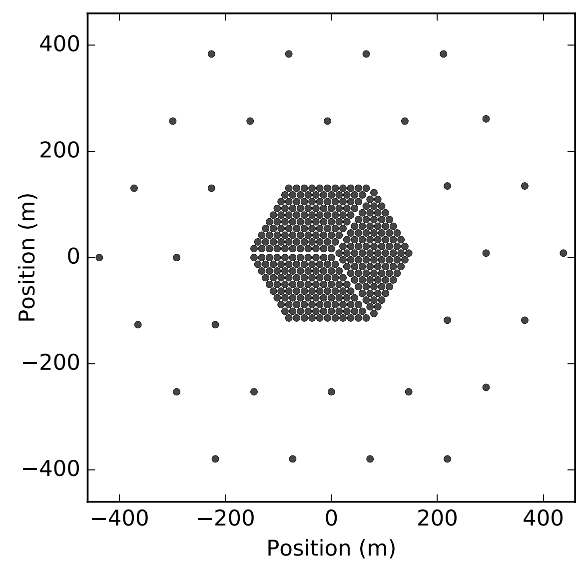
Unique Baselines:

- *Solid Hexagon:* 630
- *Split-Core:* 1501
- + Outriggers: 6140

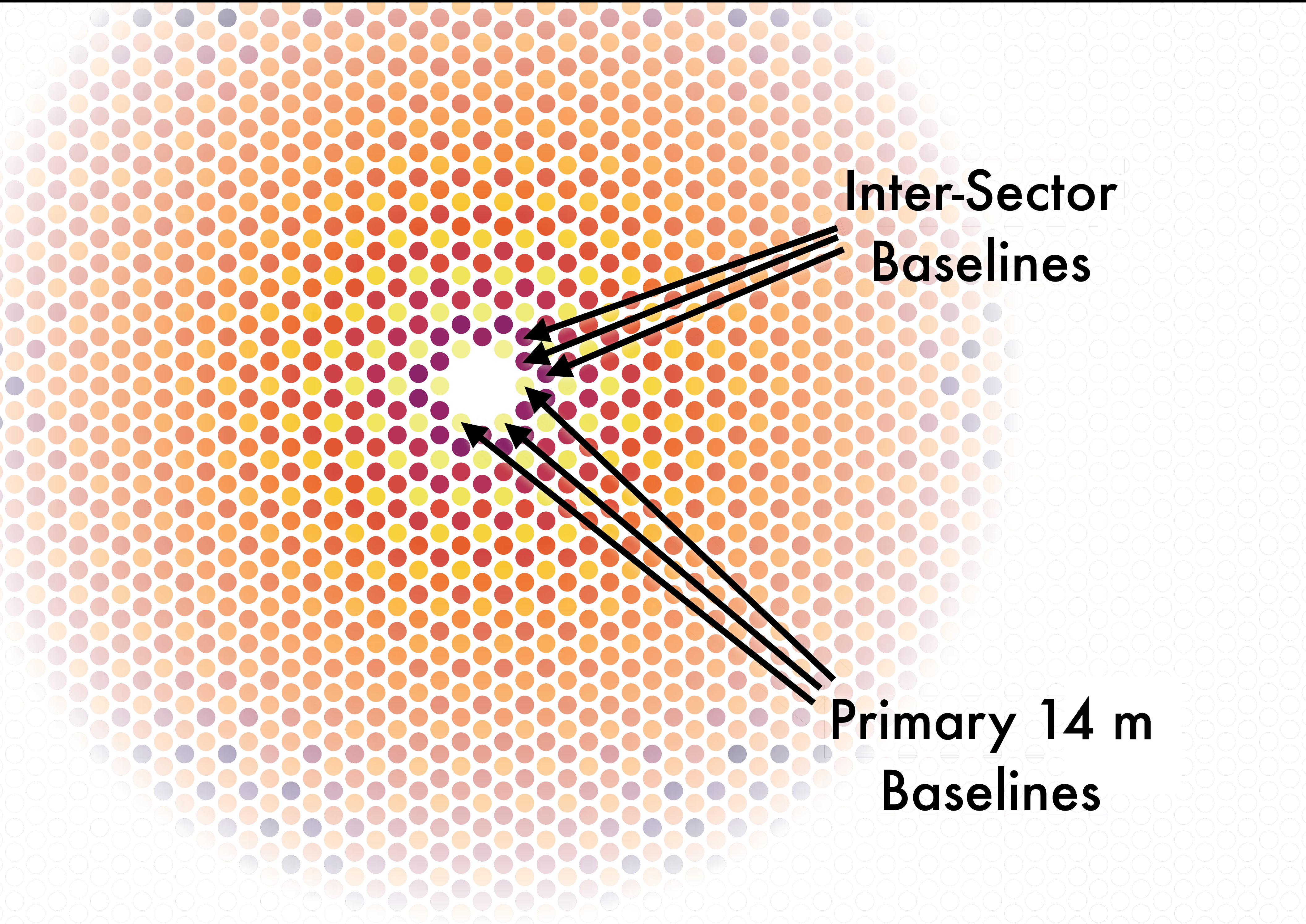
Dillon & Parsons (2016)



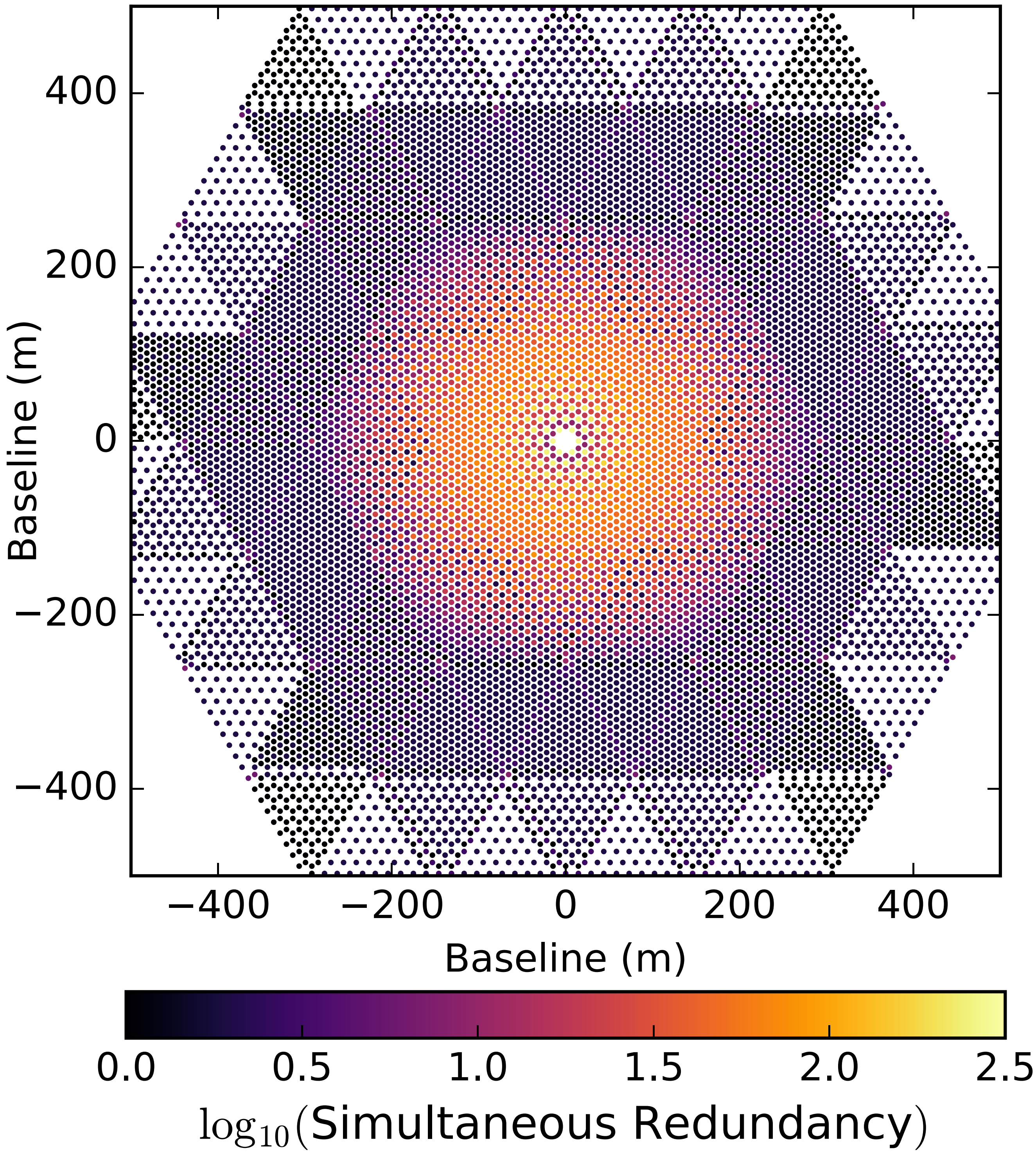
HERA's dense instantaneous coverage enables good widefield mapmaking but is still FFT-correlatable.



Dillon & Parsons (2016)



Dillon & Parsons (2016)



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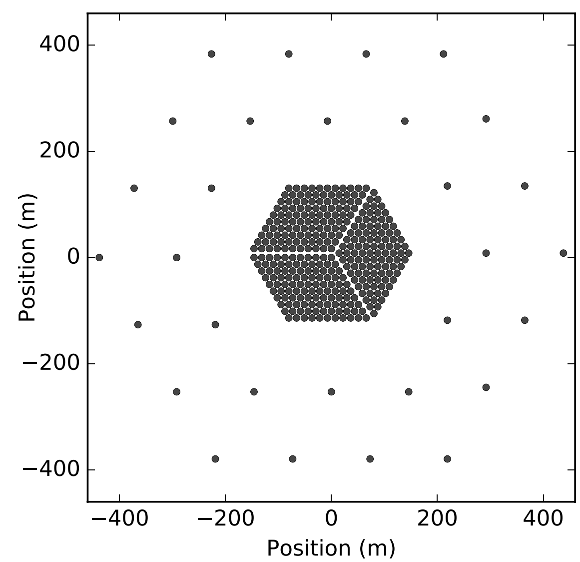
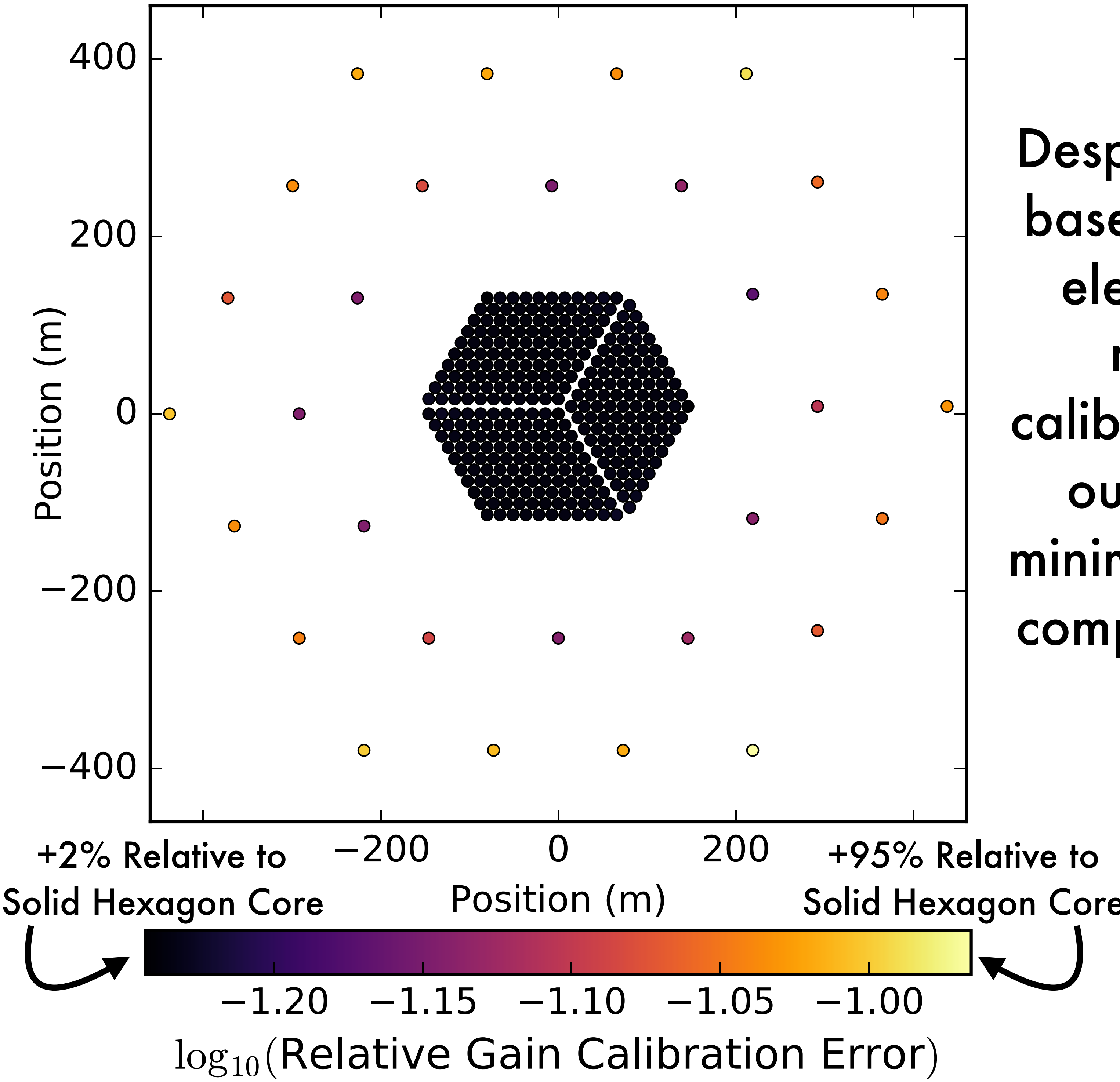


Dillon & Parsons (2016)



Despite all the extra baselines, all HERA elements can be redundantly calibrated (including outriggers) with minimal degradation compared to a solid hexagon.

Dillon & Parsons (2016)

Redundant calibration isn't quite the whole story...

$$V_{ij}^{\text{obs}}(\nu) = g_i(\nu) g_j^*(\nu) V_{ij}^{\text{true}}(\nu) + n_{ij}$$

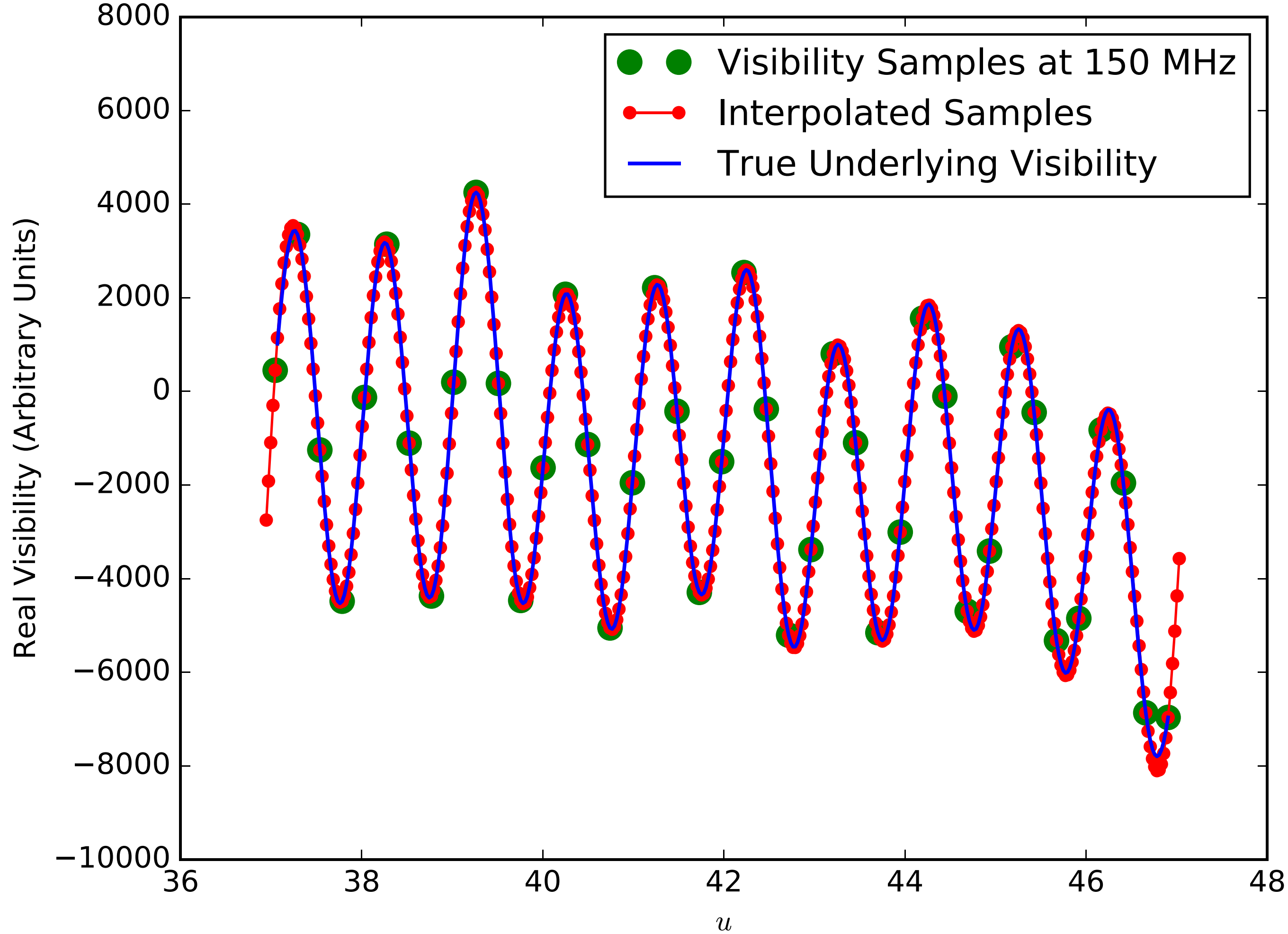
Raising all gains at one frequency is degenerate with lowering all visibilities at the same frequency.

$$V(\nu, \mathbf{b}_{ij}) = \beta(\nu) \int d^2u' \tilde{B}(\nu, \mathbf{u}' - \mathbf{u}_{ij}) \tilde{I}(\nu, \mathbf{u}')$$

Overall Bandpass:
Potentially Complicated

Fourier Sky:
Very Spectrally Smooth

Fourier interpolation gives high-u resolution visibilities with relatively few parameters.

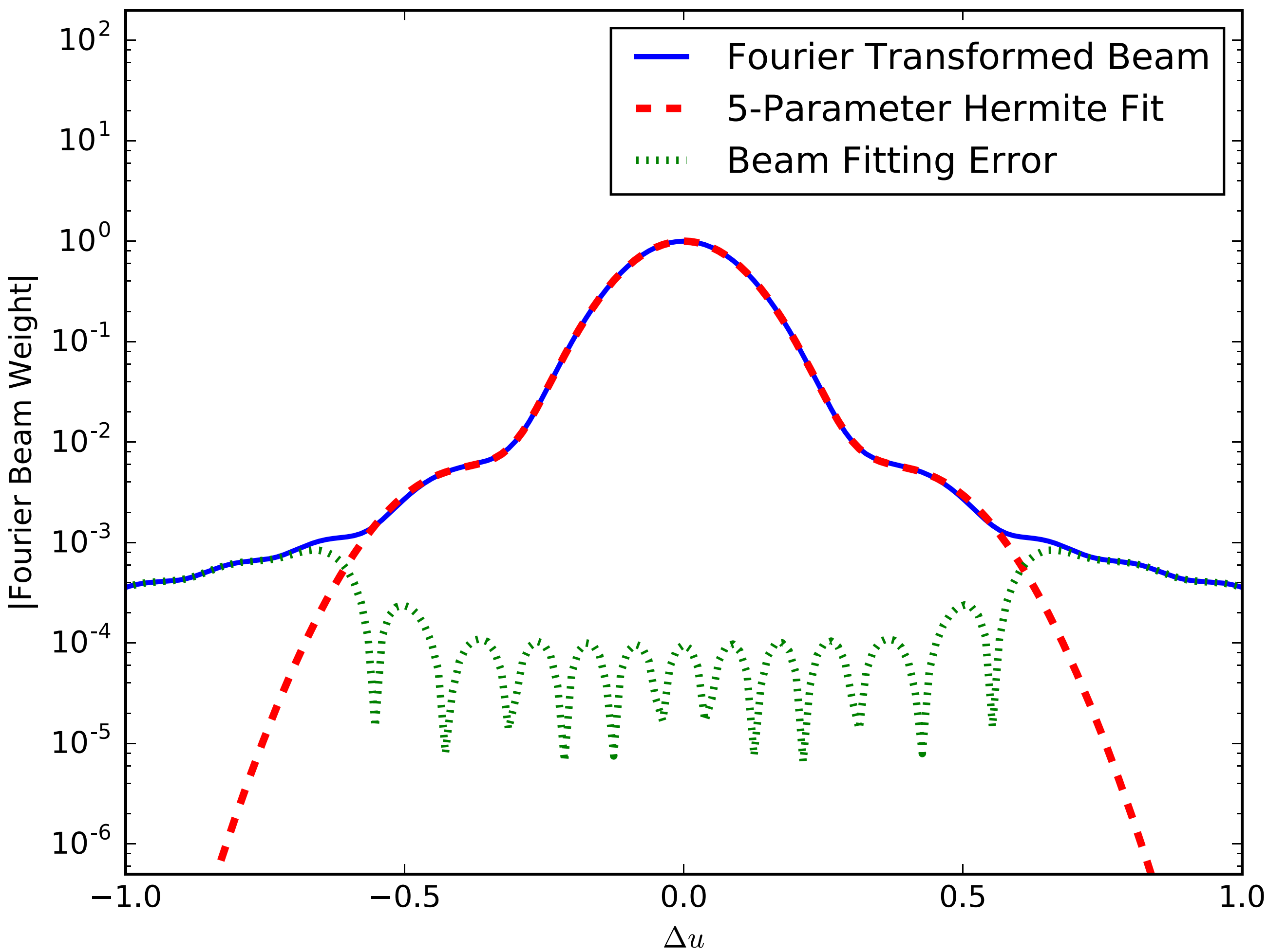


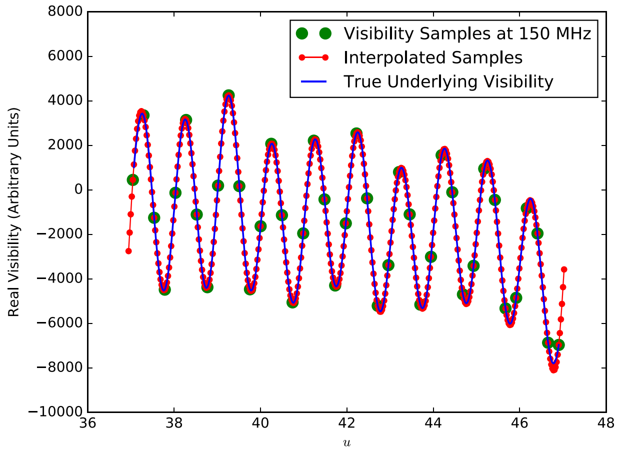
$$V(\nu, \mathbf{b}_{ij}) = \beta(\nu) \int d^2u' \tilde{B}(\nu, \mathbf{u}' - \mathbf{u}_{ij}) \tilde{I}(\nu, \mathbf{u}')$$

Overall Bandpass:
Potentially Complicated

Fourier Beam:
Fairly Spectrally Smooth

Fourier Sky:
Very Spectrally Smooth





Realistic beams are well modeled by few-parameter models.

$$V(\nu, \mathbf{b}_{ij}) = \beta(\nu) \int d^2u' \, \tilde{B}(\nu, \mathbf{u}' - \mathbf{u}_{ij}) \tilde{I}(\nu, \mathbf{u}')$$

Overall Bandpass:
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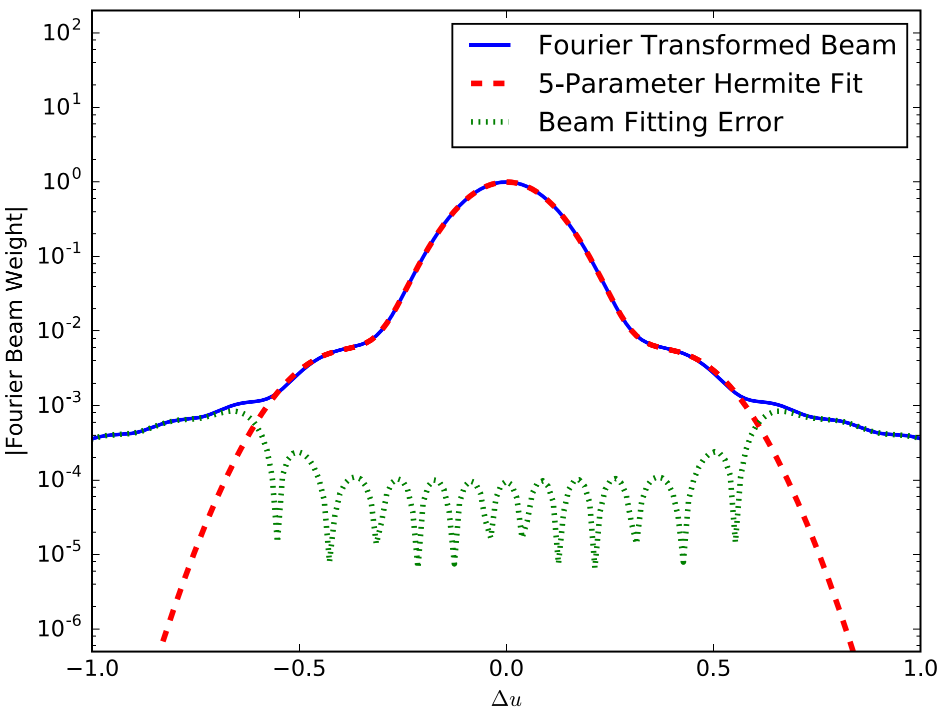
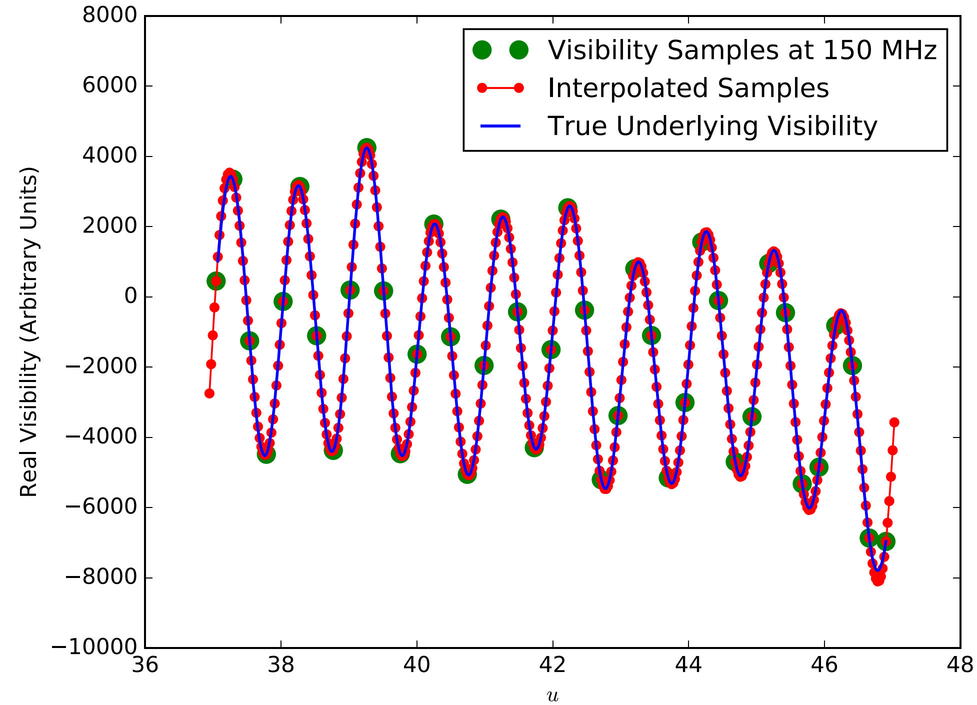


$$*$$



Convolving the two gives a faithful model for all measured visibilities.

$$V(\nu, \mathbf{b}_{ij}) = \beta(\nu) \int d^2u' \tilde{B}(\nu, \mathbf{u}' - \mathbf{u}_{ij}) \tilde{I}(\nu, \mathbf{u}')$$

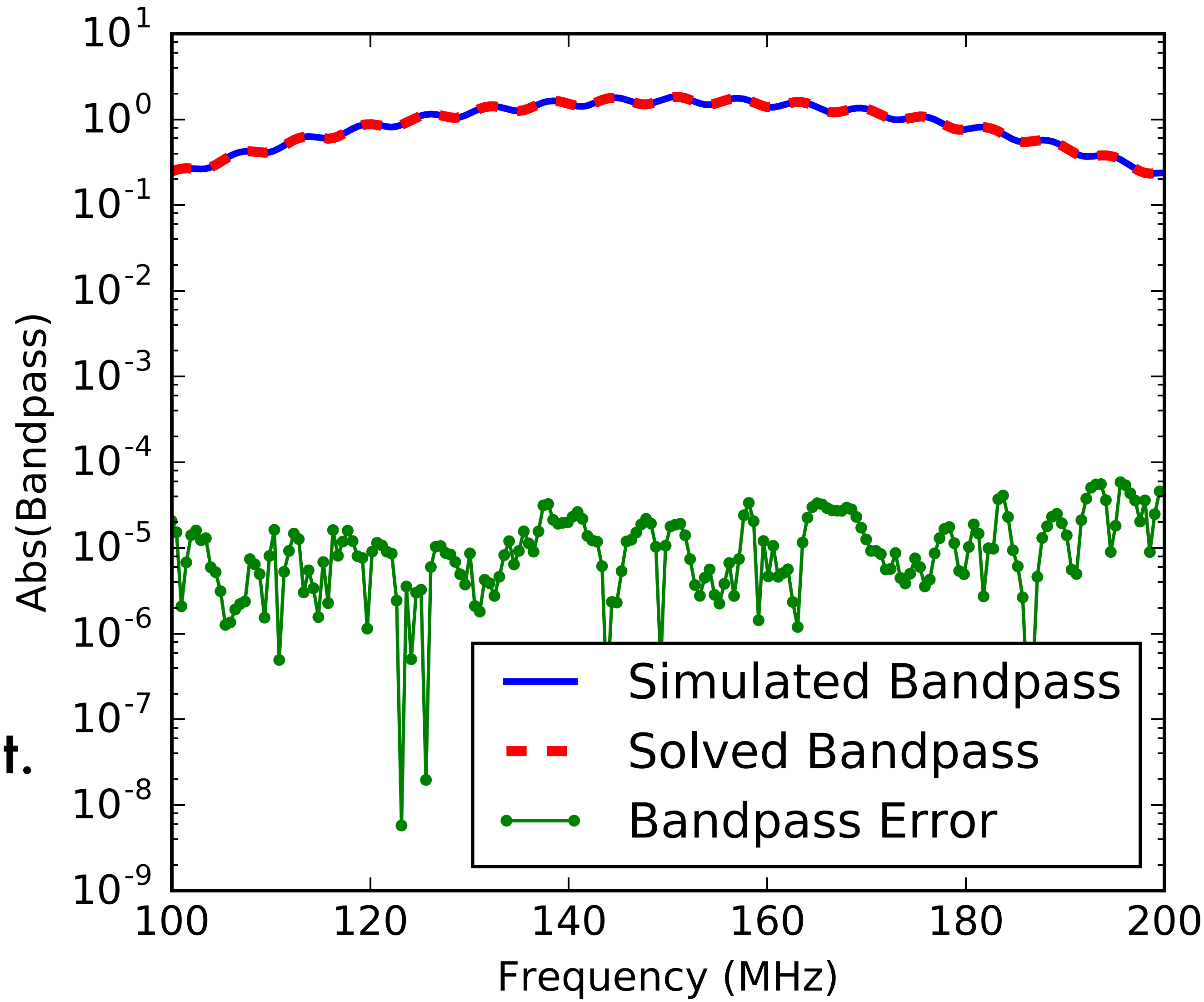
Overall Bandpass:
Potentially Complicated

Fourier Beam:
Fairly Spectrally Smooth

Fourier Sky:
Very Spectrally Smooth

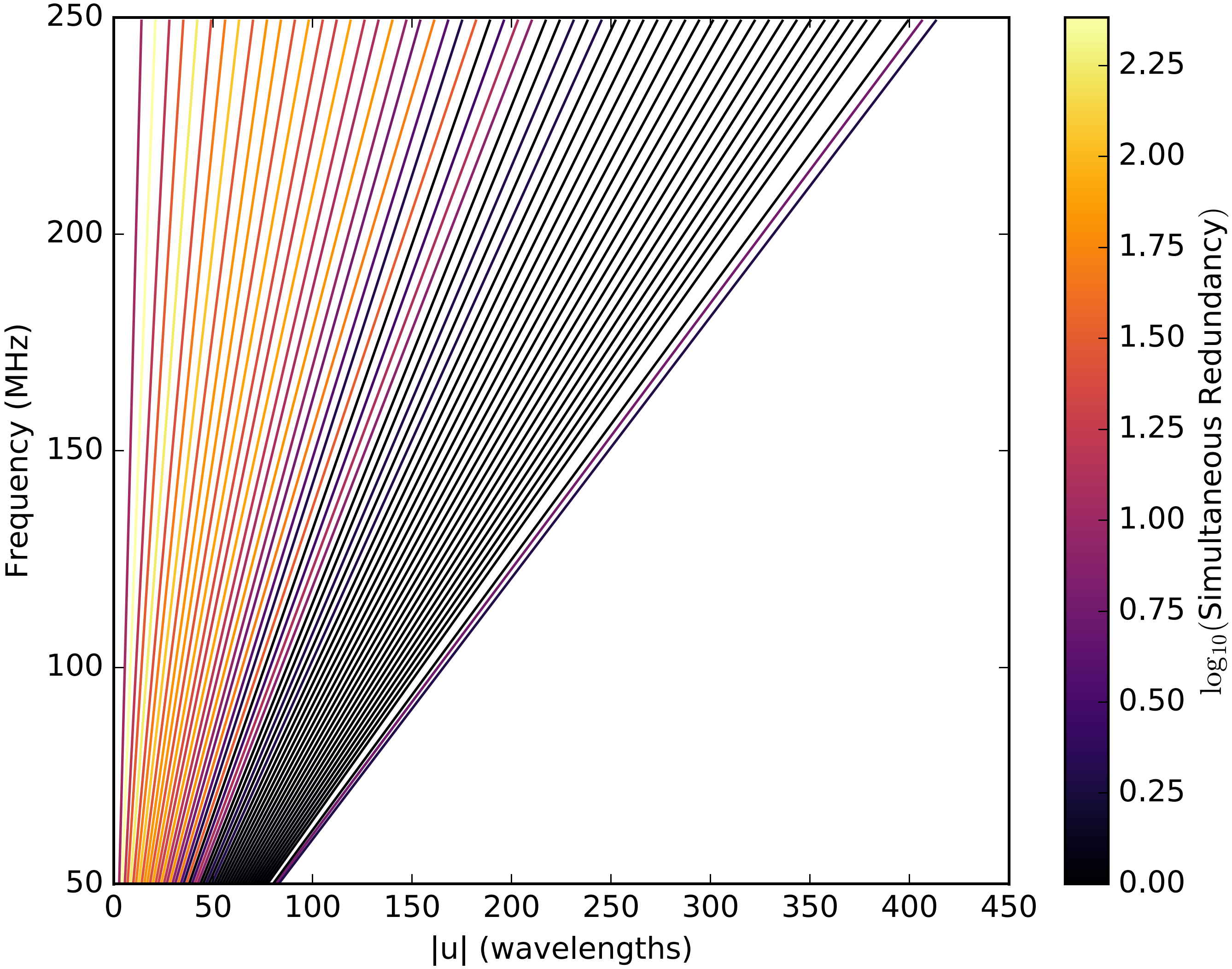
Therefore, we can use redundancy in u to calibrate the bandpass (up to some smooth function)...

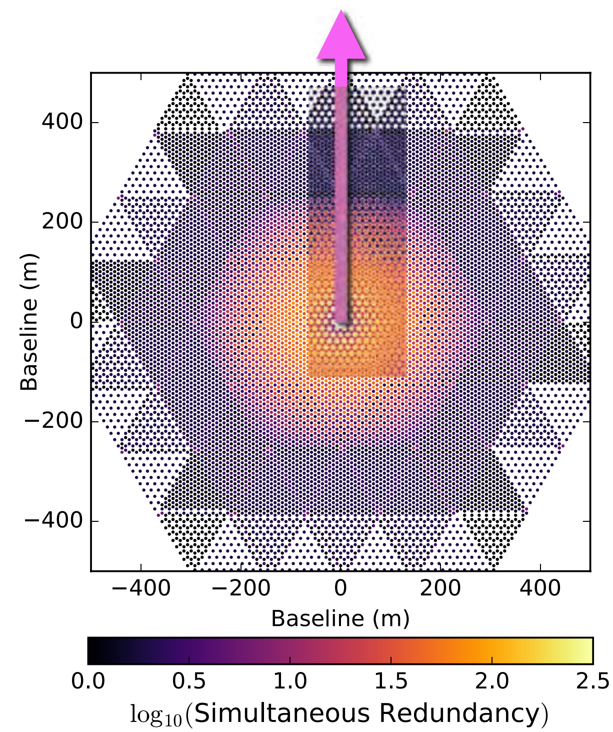
...but there's still some complications to work out.



Dillon et al. (in prep)

The split core also increases the frequency sampling at fixed (u,v), enabling better bandpass calibration.





Dillon et al. (in prep)

Precision calibration requires novel approaches.

- Sky-based calibration:
  - Modeling errors mix long and short baselines, turning the wedge into a brick.
  - This is mitigated with better weighting of modes.
- Redundant calibration:
  - Relies on redundant arrays with near-identical elements, like PAPER and HERA.
  - Normally only calibrates on a per-frequency basis, but overall bandpass calibration of PAPER and HERA is very promising.