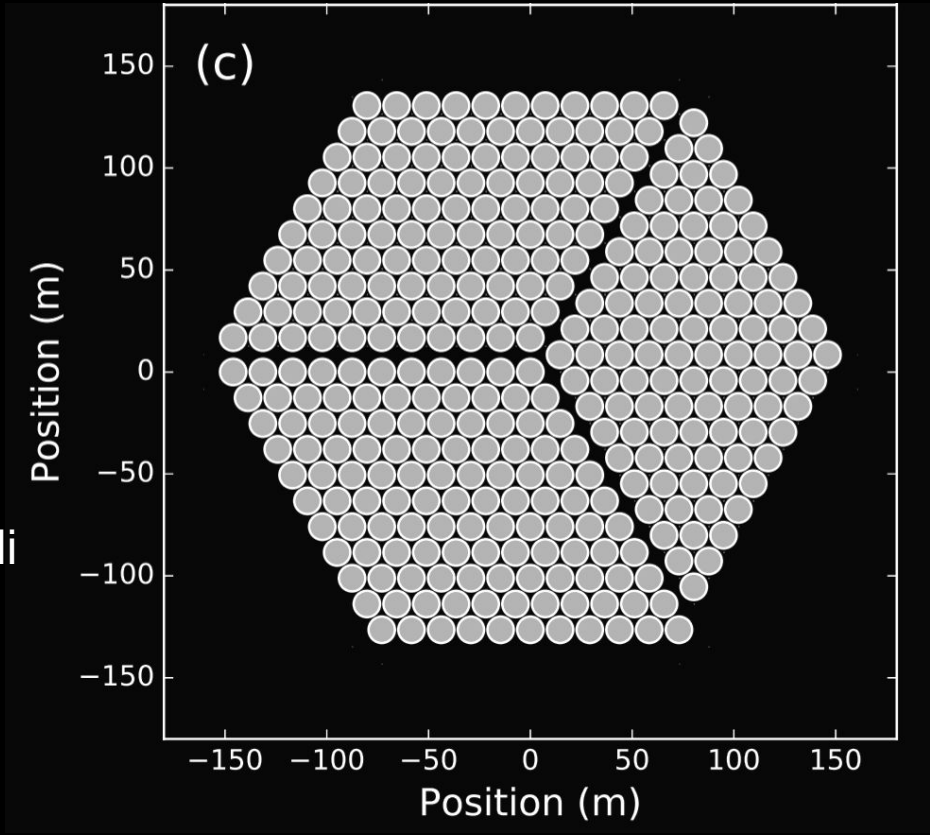
Hyperspectral FFT Imager for HERA

Deepthi Gorthi
University of California, Berkeley

Hydrogen **E**poch of **R**eionization **A**rray

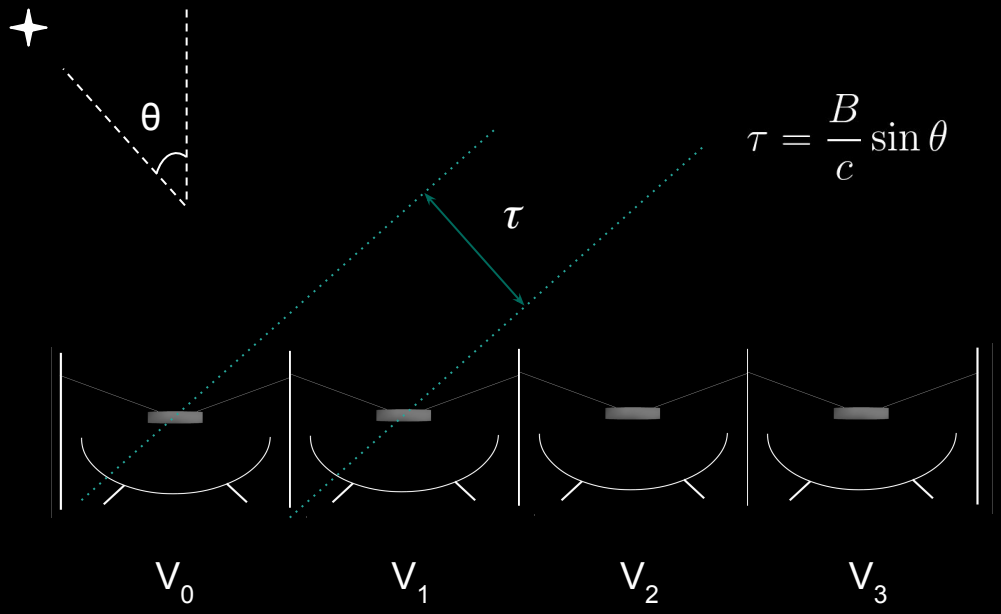
HERA

- Hexagonal grid 300m across
- 350 non steerable dishes
- Each 14m in diameter
- Built out of chicken-wire, wood and polymer pipes
- Dipole feeds being replaced by Vivaldi feeds
- BW: 50 - 250 MHz



reionization.org

Correlators become convolvers

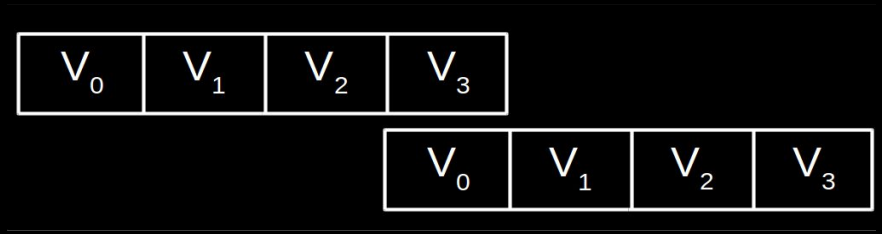


Correlator output = $V_0 * V_1$, $V_1 * V_2$, $V_2 * V_3$,

$V_0 * V_2$, $V_1 * V_3$,

$V_0 * V_3$

Correlators become convolvers

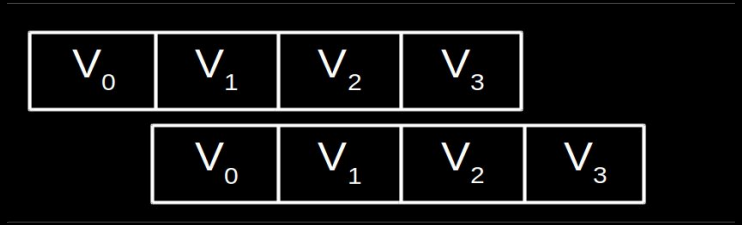


$F[3] = V_0 * V_3$

Correlators become convolvers

$F[2] = V_0 * V_2 + V_1 * V_3$

Correlators become convolvers



$$F[1] = V_0 * V_1 + V_1 * V_2 + V_2 * V_3$$

Correlators become convolvers

$F[3] = V_0 * V_3$

$F[1] = V_0 * V_1 + V_1 * V_2 + V_2 * V_3$

$F[2] = V_0 * V_2 + V_1 * V_3$

$F[0] = V_0 * V_0 + V_1 * V_1 + V_2 * V_2 + V_3 * V_3$

Convolution:

$$F_i = \Sigma_{n=i}^{N-1} V_n V_{n-i}^*$$

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$$F_i = \Sigma_{n=i}^{N-1} V_n V_{n-i}^*$$

Redefine

$V_n =$

V_0	V_1	V_2	V_3	0	0	0	0

$$F_i = \Sigma_{n=0}^{N-1} V_n V_{n-i}^* = V_n * V_{-n}^*$$

Use convolution theorem instead?

Convolution:

$$F_i = \Sigma_{n=i}^{N-1} V_n V_{n-i}^*$$

Redefine

$V_n =$

V_0	V_1	V_2	V_3	0	0	0	0

$$F_i = \Sigma_{n=0}^{N-1} V_n V_{n-i}^* = V_n * V_{-n}^*$$

Applying the Convolution Theorem

$$\tilde{F} = \tilde{V}_n \tilde{V}_{-n}^* = |\tilde{V}_n|^2$$

Use convolution theorem instead?

Number of Computations for N antennas in the array:

Convolution: $F_i = \Sigma_{n=0}^{N-1} V_n V_{n-i}^*$

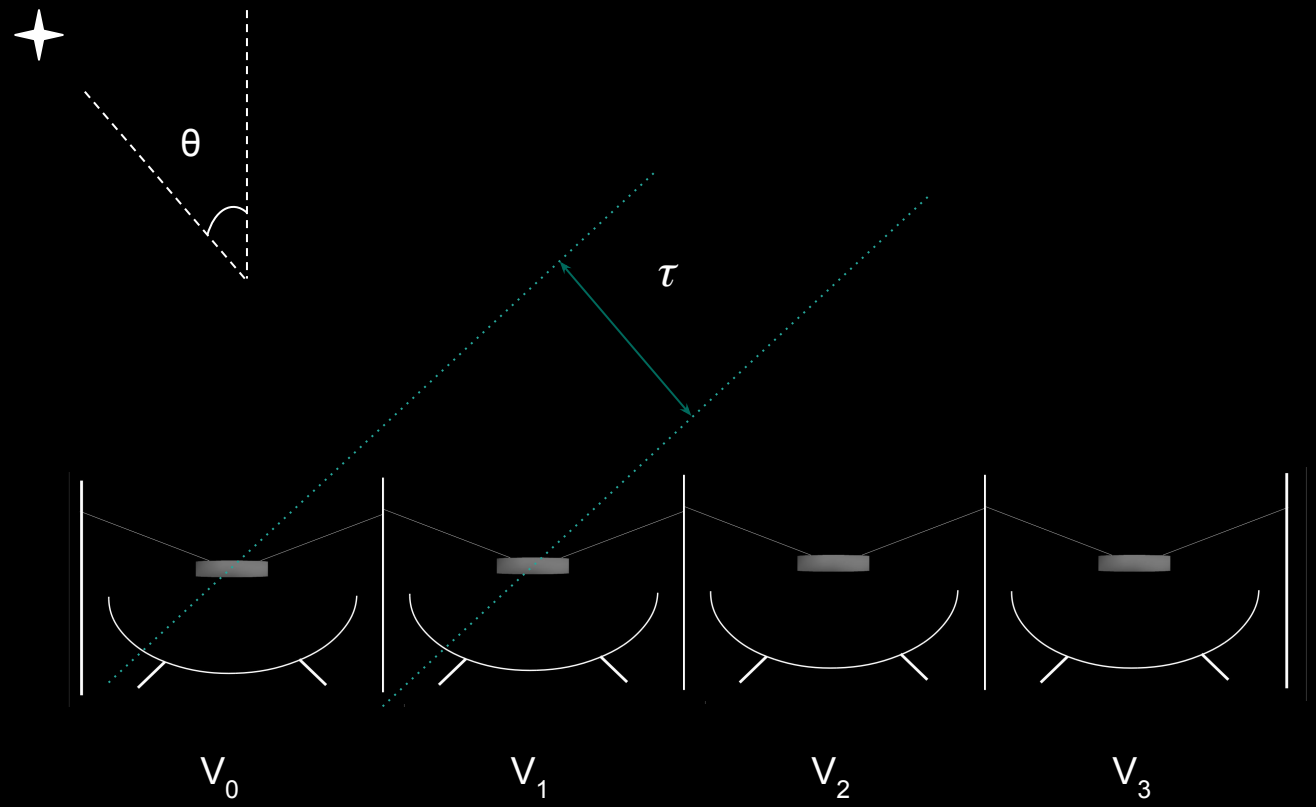
$$\frac{N(N-1)}{2} \sim N^2$$

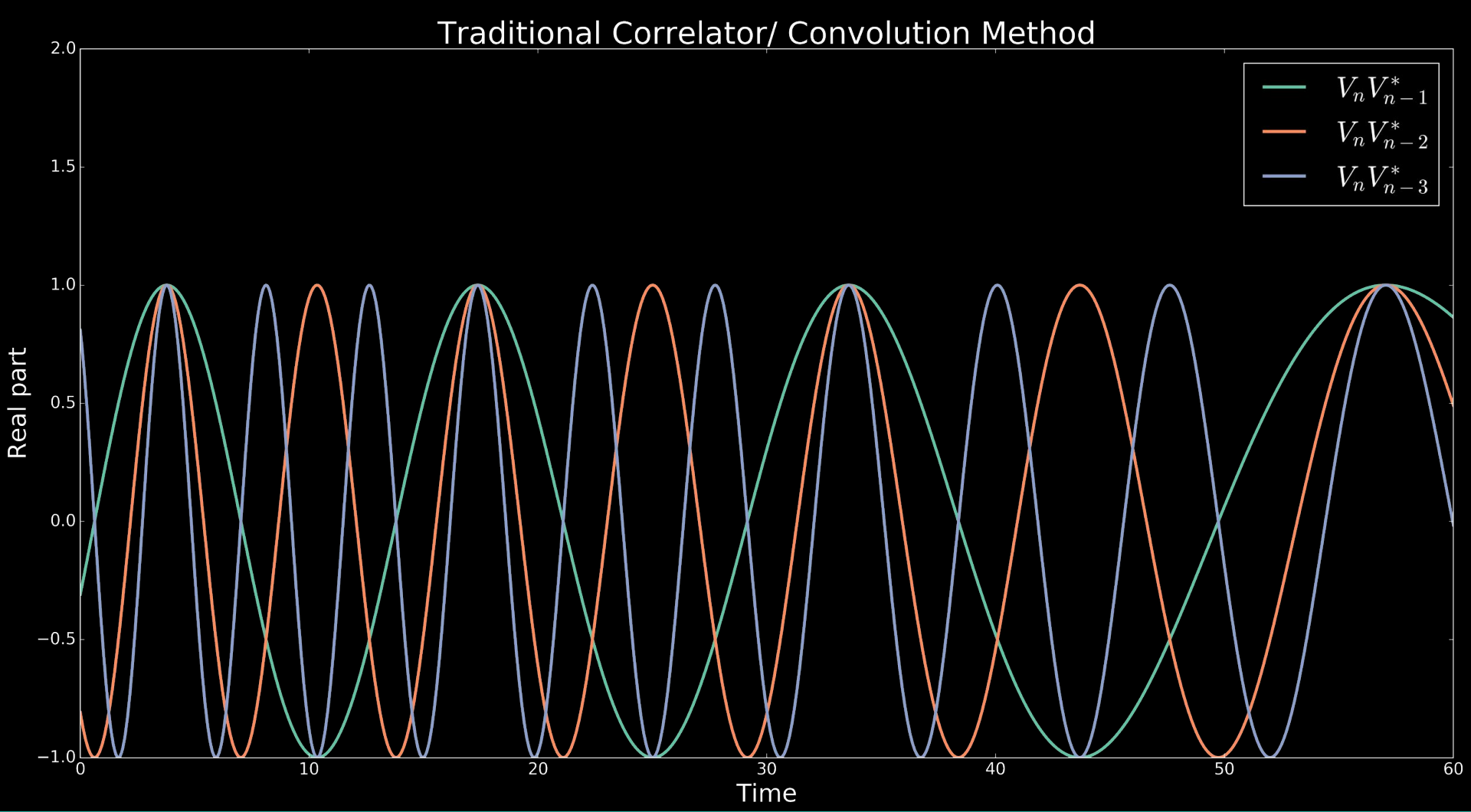
Redefine $V_n =$

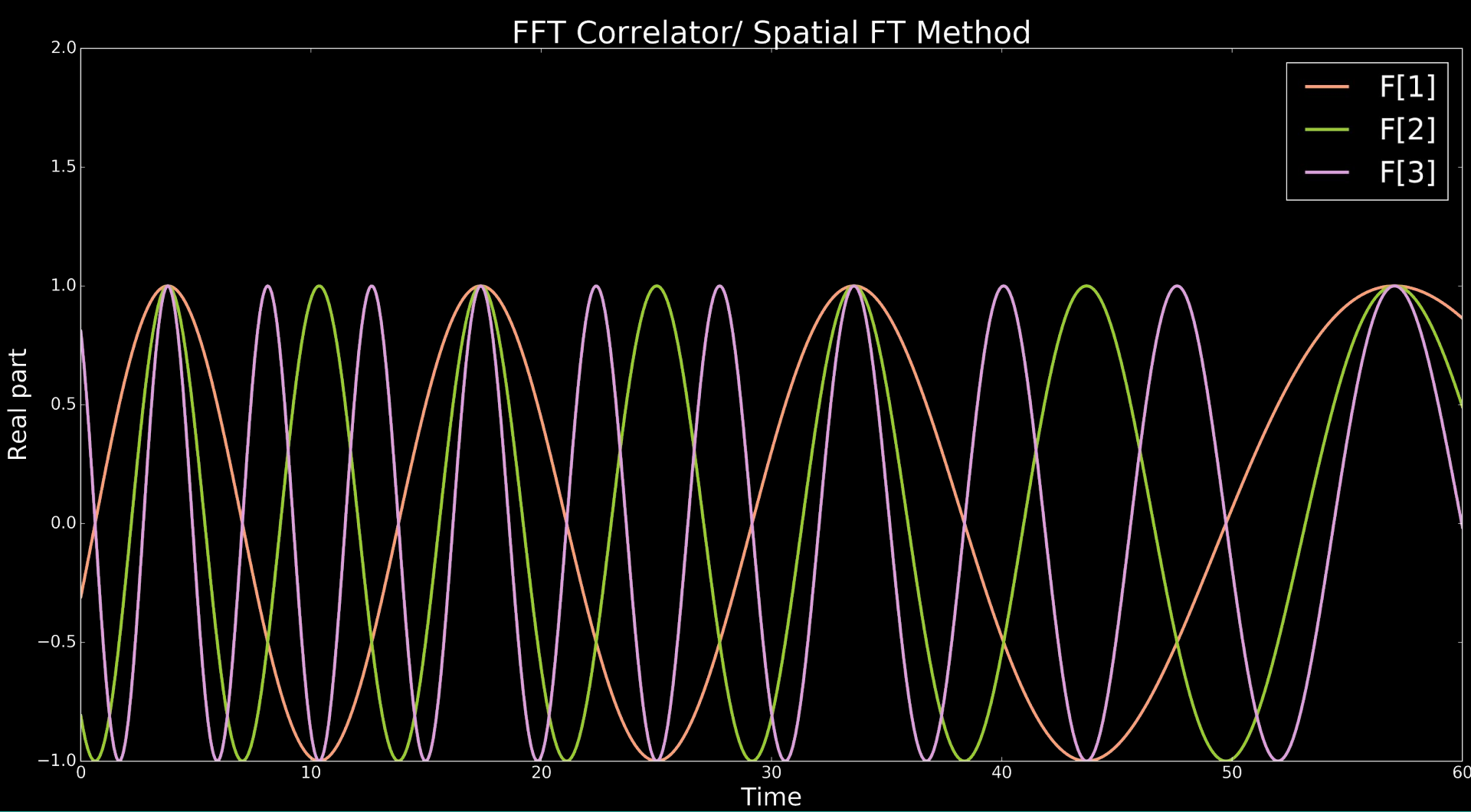
V_0	V_1	V_2	V_3	0	0	0	0

FFT $\tilde{F} = \tilde{V_n} \tilde{V_{-n}^*} = |\tilde{V_n}|^2$

$$\sim 2N \log(2N)$$


θ
τ
V0
V1
V2
V3

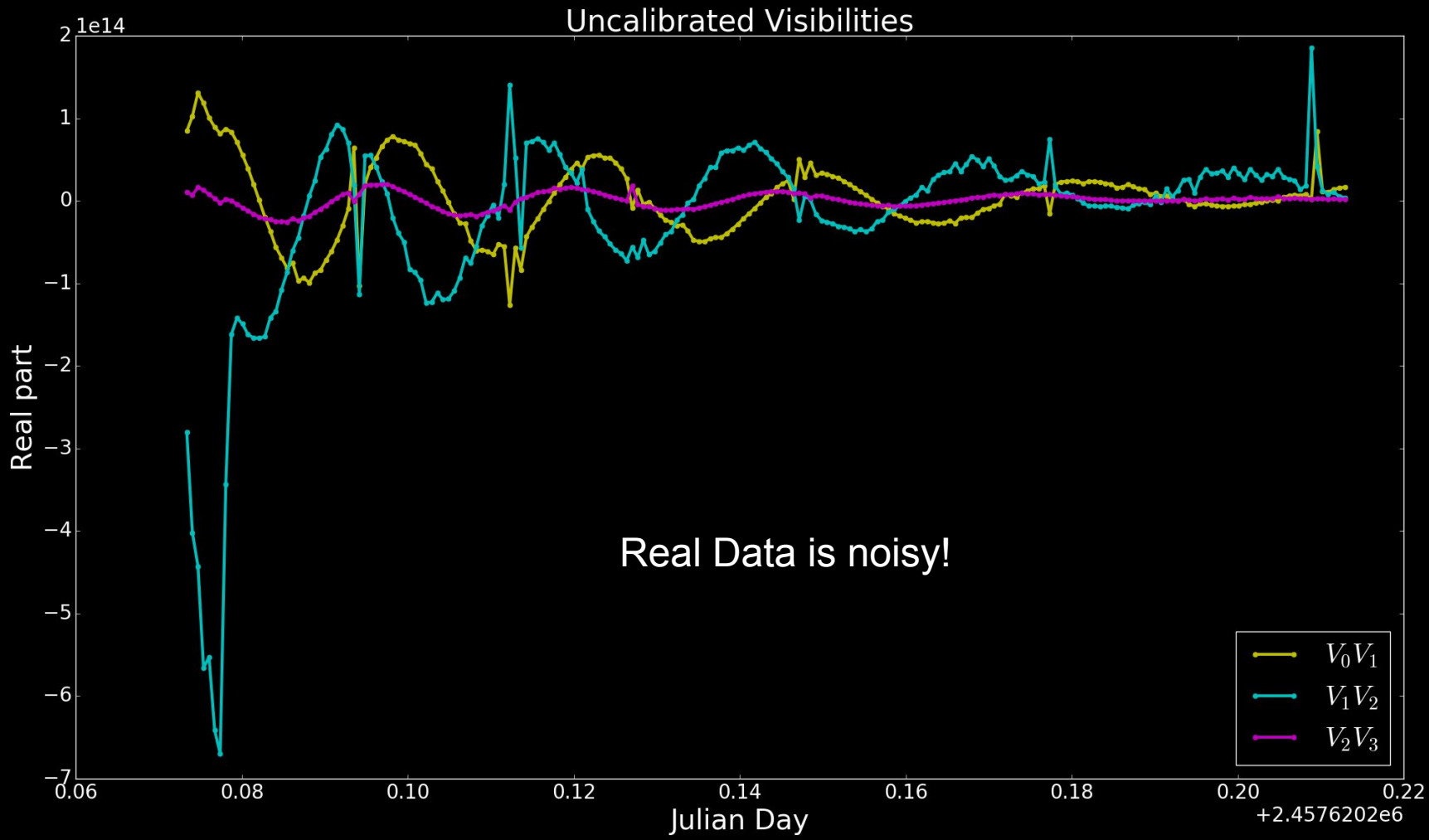

Traditional Correlator/ Convolution Method
$V_n V_{n-1}^*$
$V_n V_{n-2}^*$
$V_n V_{n-3}^*$
Real part
Time
2.0
1.5
1.0
0.5
0.0
−0.5
−1.0
0
10
20
30
40
50
60

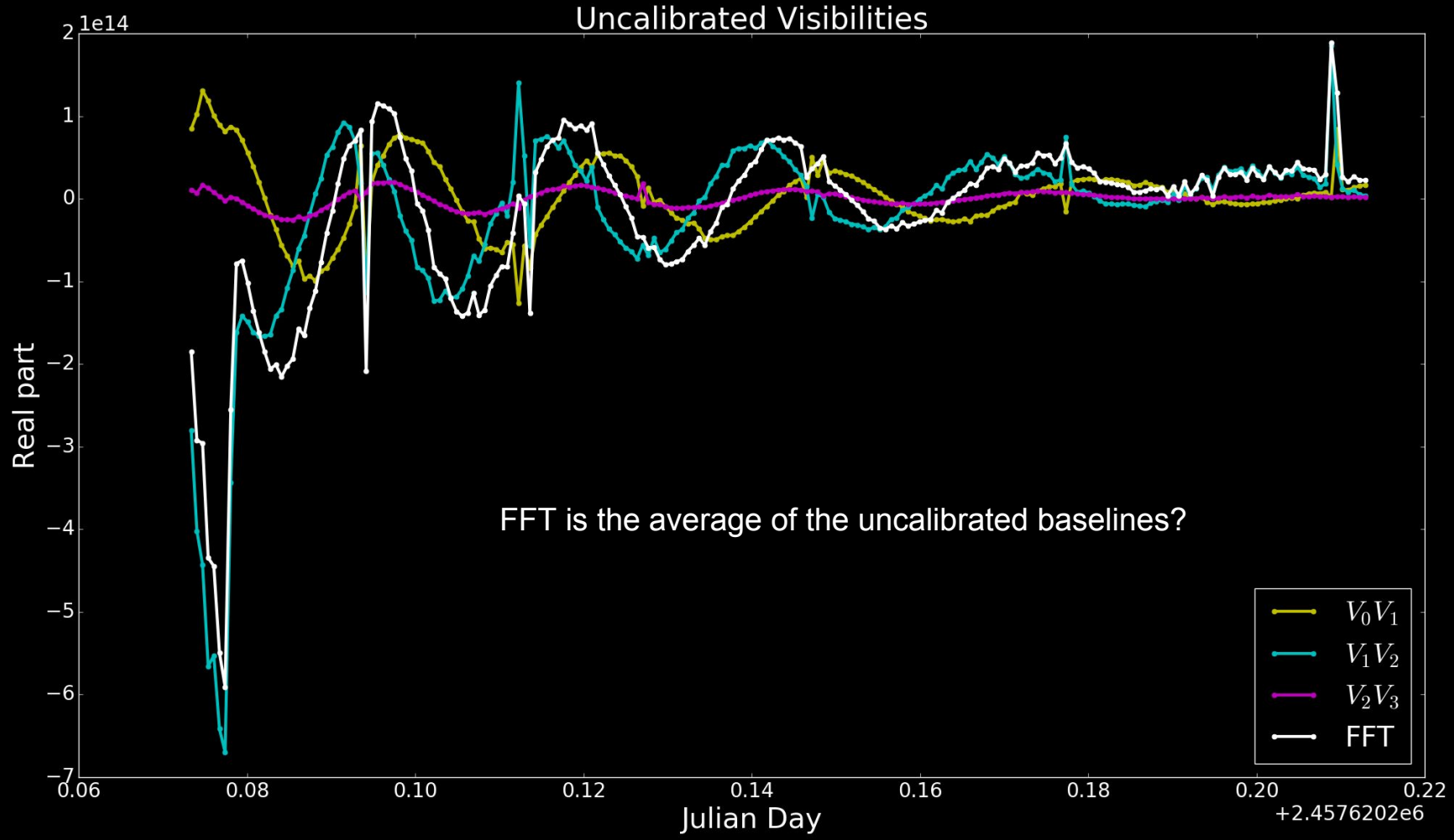

FFT Correlator/ Spatial FT Method
F[1]
F[2]
F[3]
Real part
Time
2.0
1.5
1.0
0.5
0.0
−0.5
−1.0
0
10
20
30
40
50
60

Motivation

- Enable HERA and HERA-like arrays for radio transient observations.
  - Sensitivity of Arecibo
  - Much larger FOV

- Prepare for HERA-3
  - >1000 antennas


Uncalibrated Visibilities
1e14
2
1
0
−1
−2
−3
−4
−5
−6
−7
Real part
0.06
0.08
0.10
0.12
0.14
0.16
0.18
0.20
0.22
Julian Day
+2.4576202e6
Real Data is noisy!
V_0V_1
V_1V_2
V_2V_3


Uncalibrated Visibilities
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−4
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Real part
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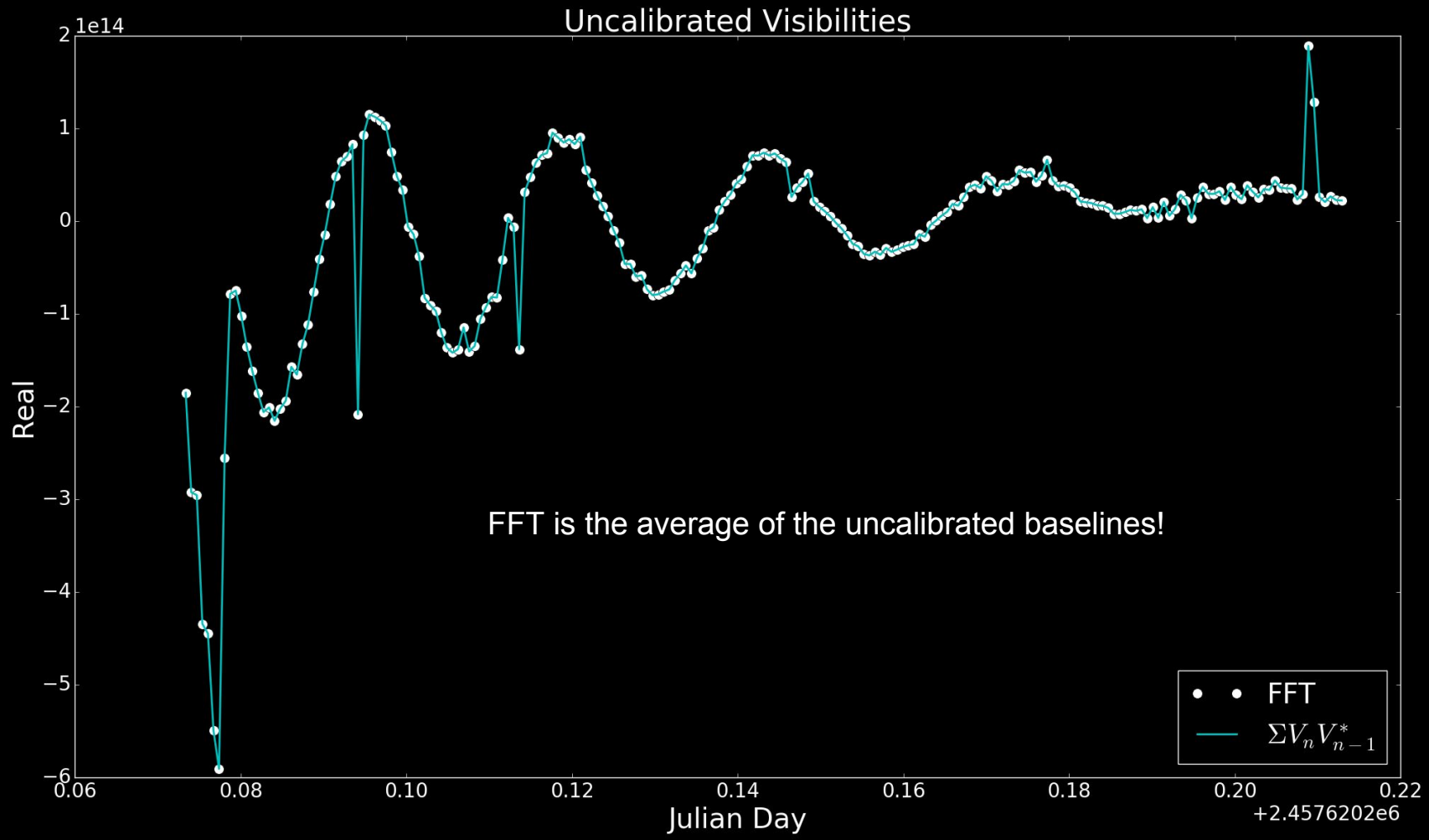
Julian Day
+2.4576202e6
FFT is the average of the uncalibrated baselines?
V_0V_1
V_1V_2
V_2V_3
FFT

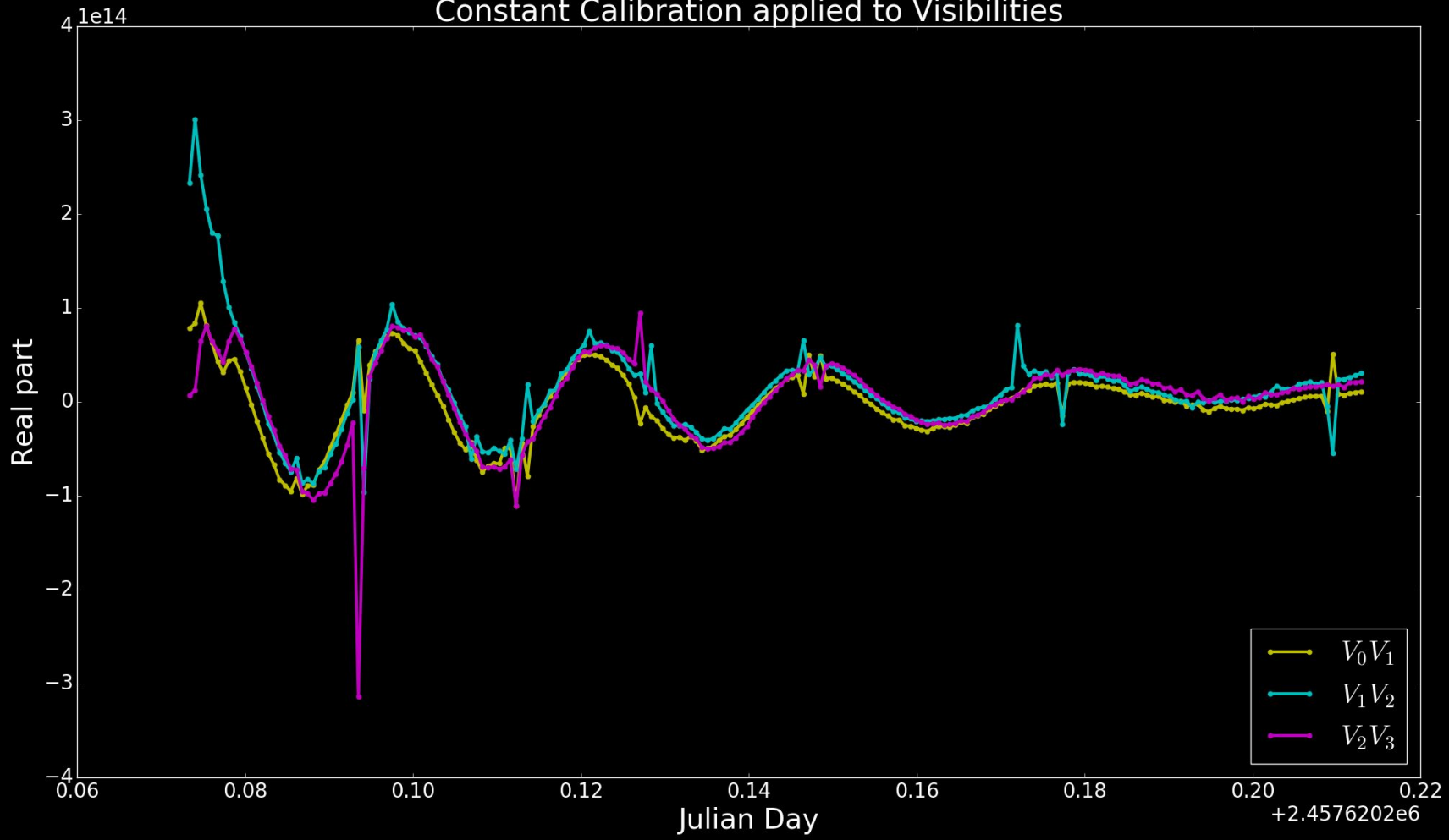

Uncalibrated Visibilities
1e14
Real
Julian Day
+2.4576202e6
FFT is the average of the uncalibrated baselines!
FFT
$\Sigma V_n V_{n-1}^*$

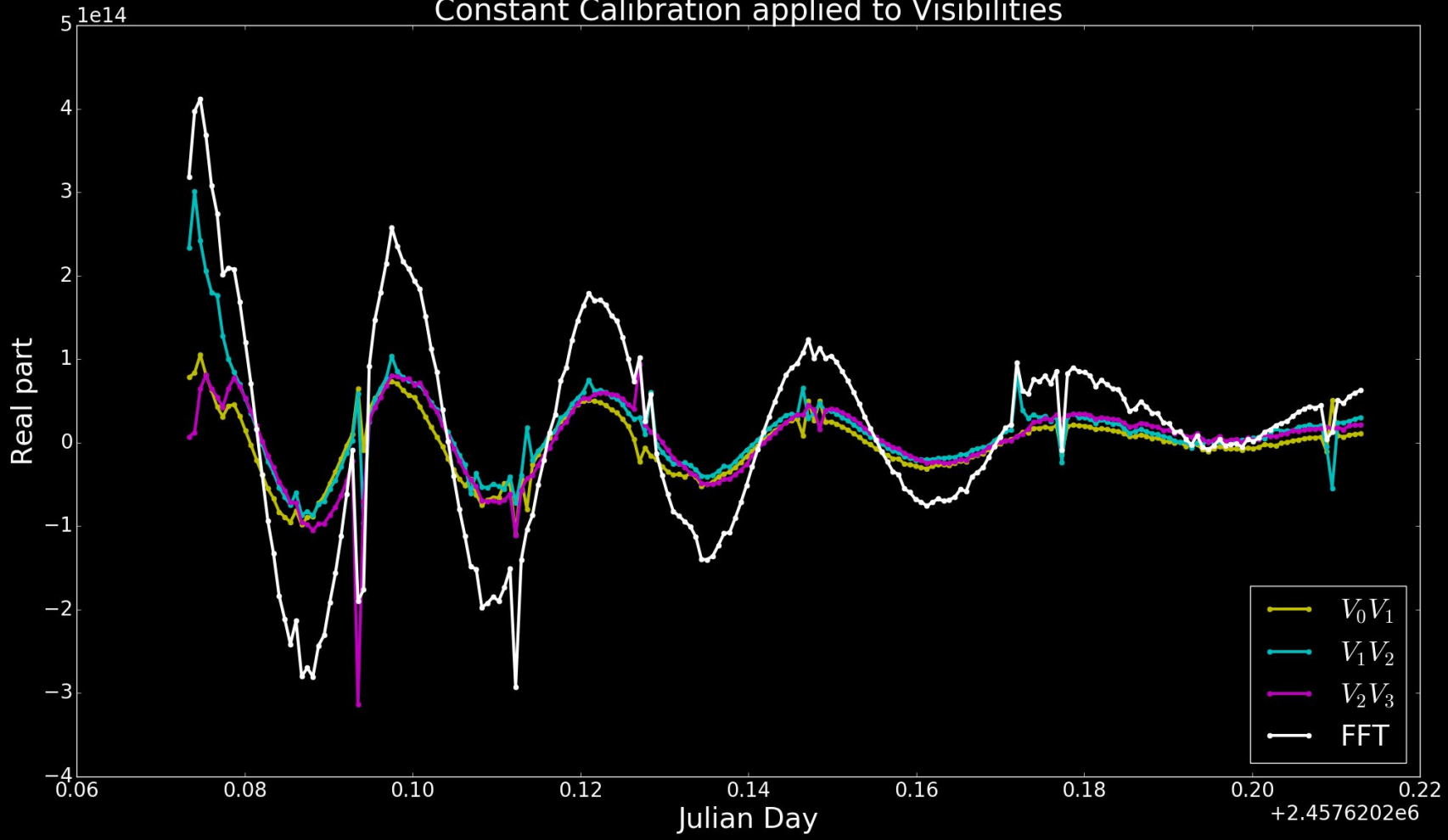
Calibrate the visibilities

$$V_{M,10} = g_1\, g_0^*\, V_{T,1}$$
$$V_{M,21} = g_2\, g_1^*\, V_{T,1}$$
$$V_{M,32} = g_3\, g_2^*\, V_{T,1}$$

$$V_{M,20} = g_2\, g_0^*\, V_{T,2}$$
$$V_{M,31} = g_3\, g_1^*\, V_{T,2}$$

5 equations in 6 variables- solve up to a degeneracy in 2 variables. For the multiple calibration methods see Liu et al (2010).


Constant Calibration applied to Visibilities
1e14
4
3
2
1
0
−1
−2
−3
−4
Real part
0.06
0.08
0.10
0.12
0.14
0.16
0.18
0.20
0.22
Julian Day
+2.4576202e6
V_0V_1
V_1V_2
V_2V_3


Constant Calibration applied to Visibilities
1e14
5
4
3
2
1
0
−1
−2
−3
−4
Real part
0.06
0.08
0.10
0.12
0.14
0.16
0.18
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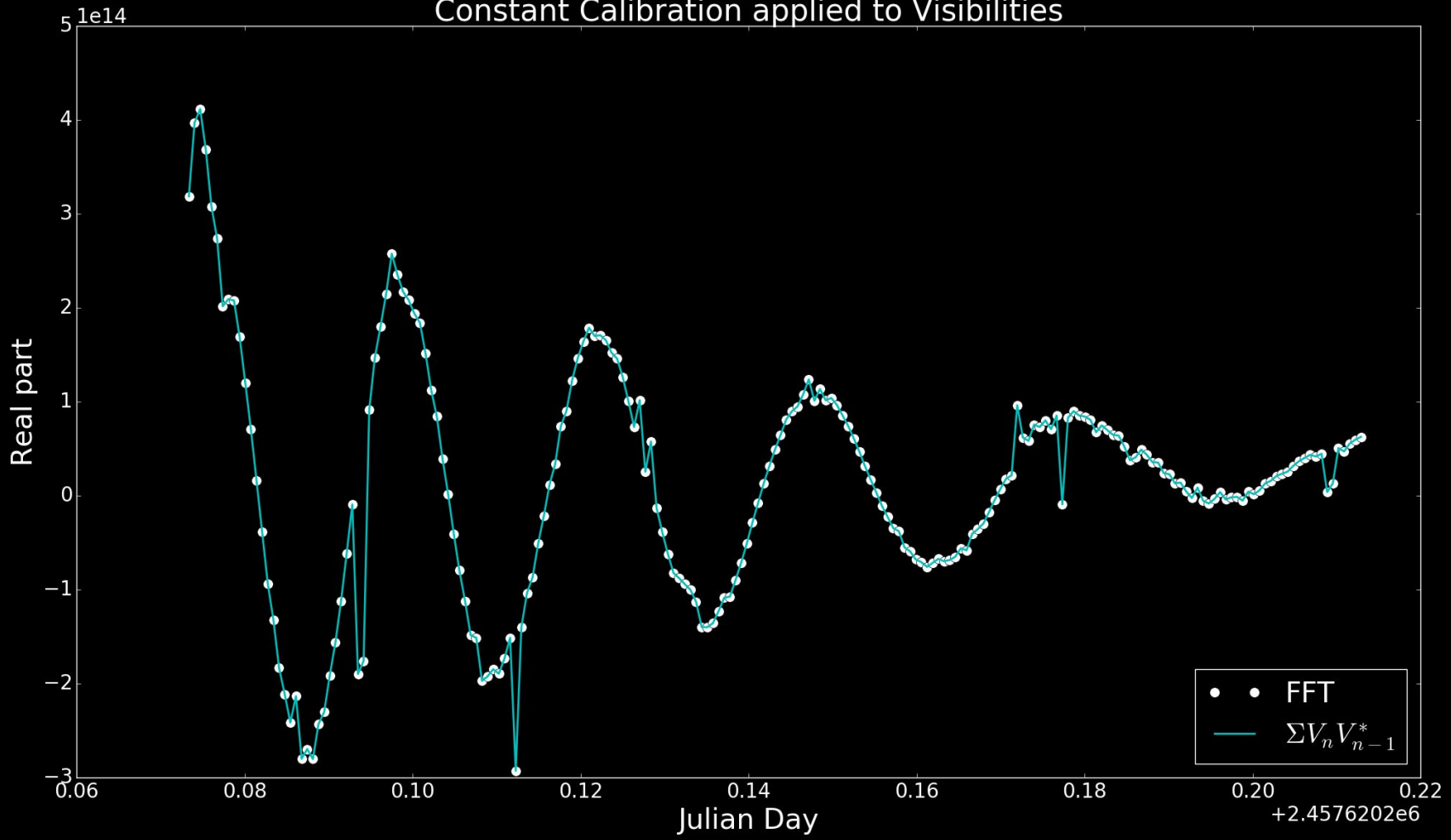
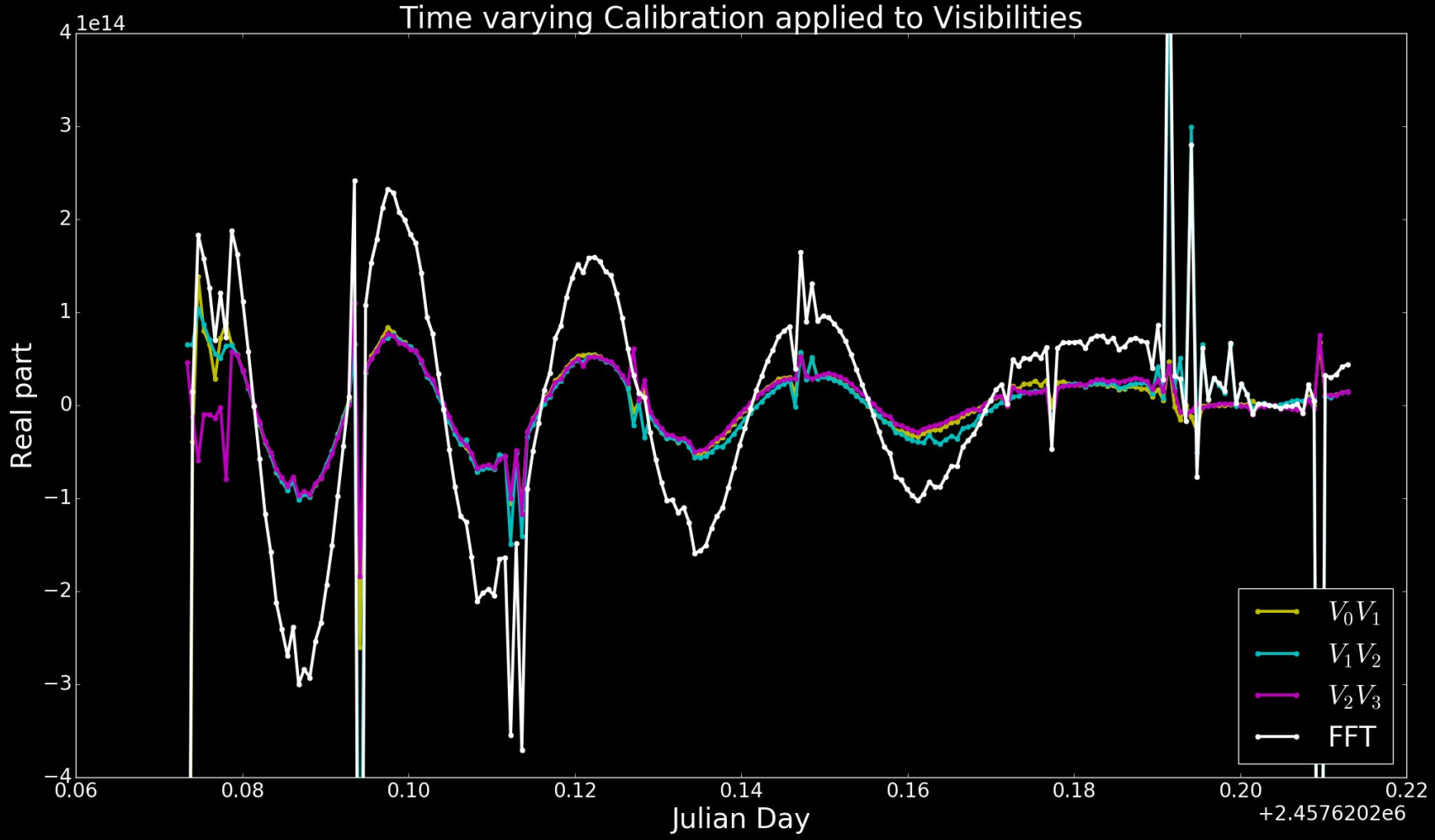
0.22
+2.4576202e6
Julian Day
V_0V_1
V_1V_2
V_2V_3
FFT


Constant Calibration applied to Visibilities
1e14
5
4
3
2
1
0
−1
−2
−3
Real part
0.06
0.08
0.10
0.12
0.14
0.16
0.18
0.20
0.22
+2.4576202e6
Julian Day
FFT
$\Sigma V_n V_{n-1}^*$


Time varying Calibration applied to Visibilities
1e14
4
3
2
1
0
−1
−2
−3
−4
Real part
0.06
0.08
0.10
0.12
0.14
0.16
0.18
0.20
0.22
Julian Day
+2.4576202e6
V_0V_1
V_1V_2
V_2V_3
FFT

Future Work

- Can one calibrate antenna phase and gain from just voltages- without computing visibilities?
- Calibration in real time during the spatial FFT step.
- Minimum number of visibilities required to calibrate “enough” antennas.
- What level of calibration is required for EoR science with ~1000 antennas.