A Resistive Wideband Space Beam Splitter

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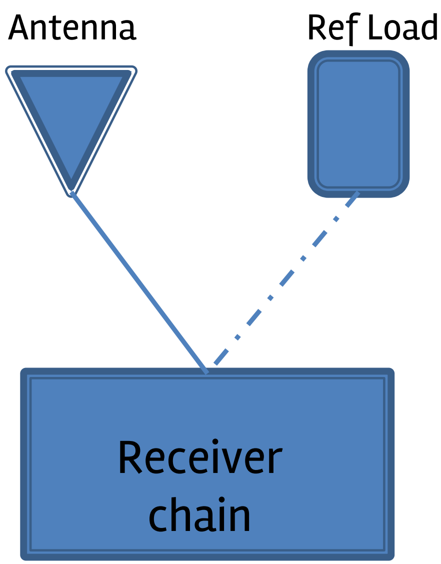
Motivation

- The high redshift 21 cm signal studies can give us useful insights into the structure formation and birth of the first stars.
- The signal of interest is orders of magnitude fainter than the galactic and extragalactic foregrounds.
- Foreground subtraction is limited by frequency-dependent coupling of radiation by the instruments.
- Specially designed radiometers are required to reduce these systematics of the instruments and generate minimal noise.
- ***Hence the idea of Zero baseline Interferometer using a Beam splitter***

Schemes to overcome current limitations:

➢ Delay switches



Disadvantages:

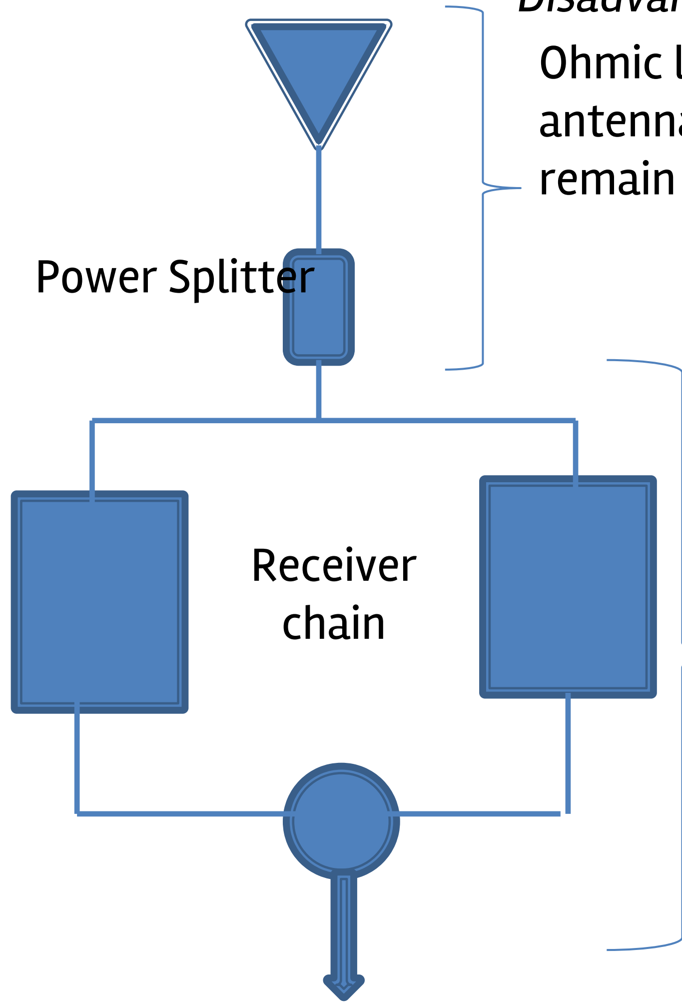
Impedance matching while load switching

Multipath noise propagation

➢ Cross Correlation

Disadvantages:

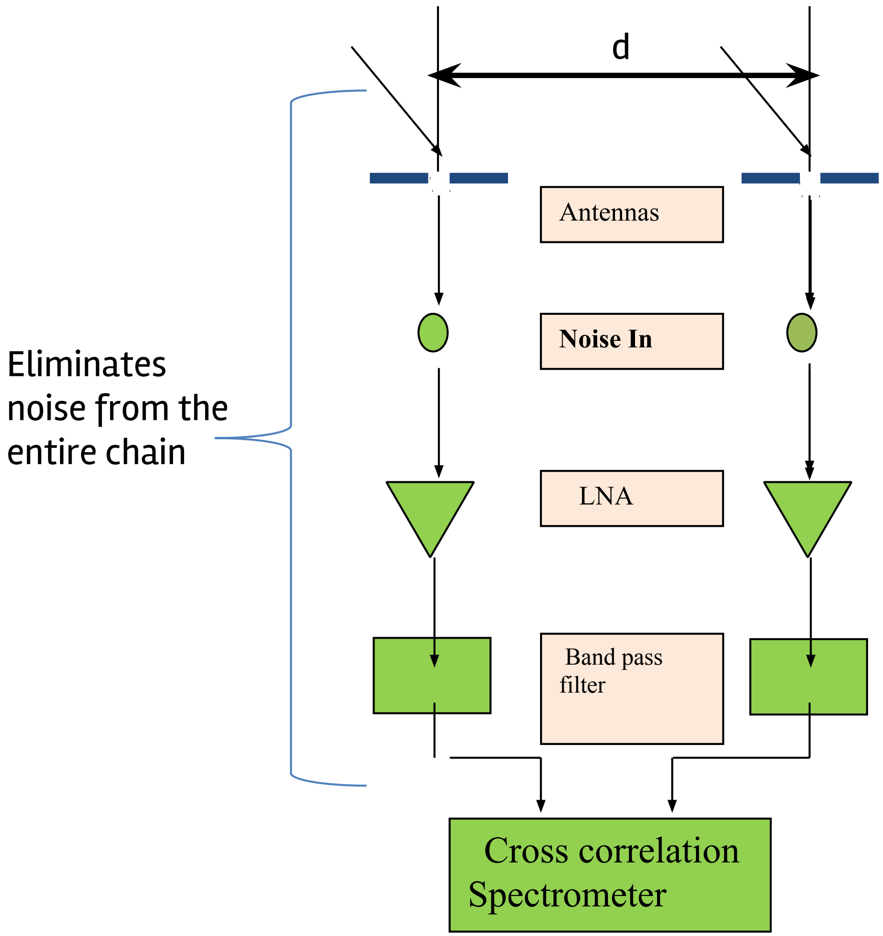
Ohmic losses from antenna and balun remain



Noise cancels – Statistically independent

Radio interferometers

Insofar, the best way to eliminate all the possible receiver noises seen in the previous two



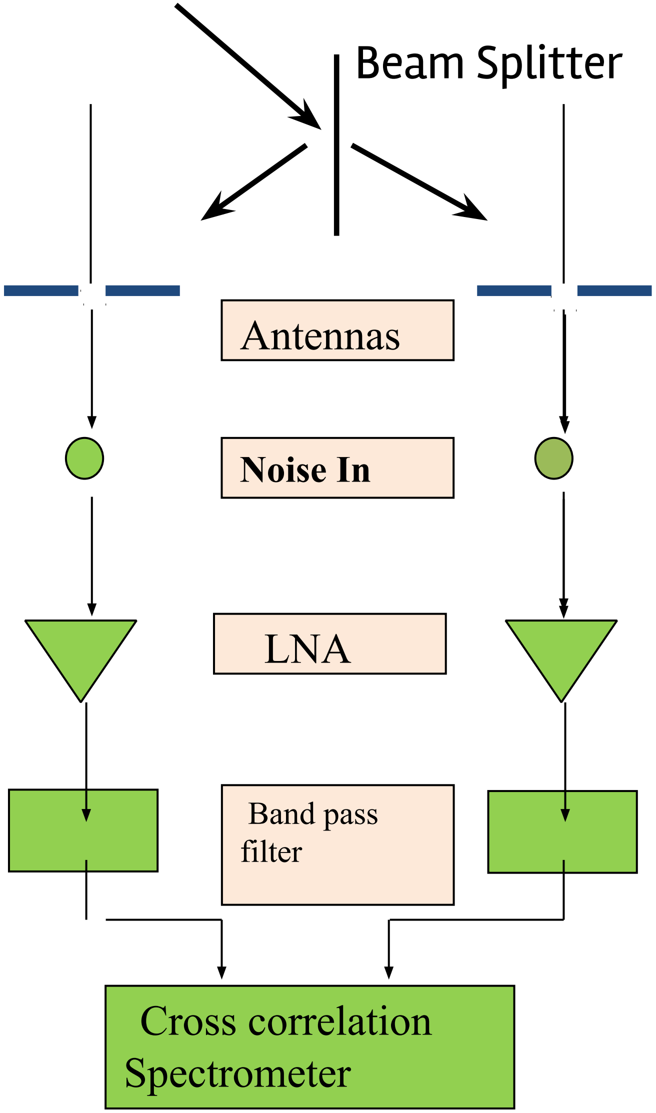
'd' determines the resolution seen on the sky

Greater, the finer. Smaller, the larger on the sky.

In effect with any non-zero value of 'd', the interferometer remains almost **blind to the uniform brightness** of sky

Ideally, d = 0 is needed to get the common mode power

The answer is ……

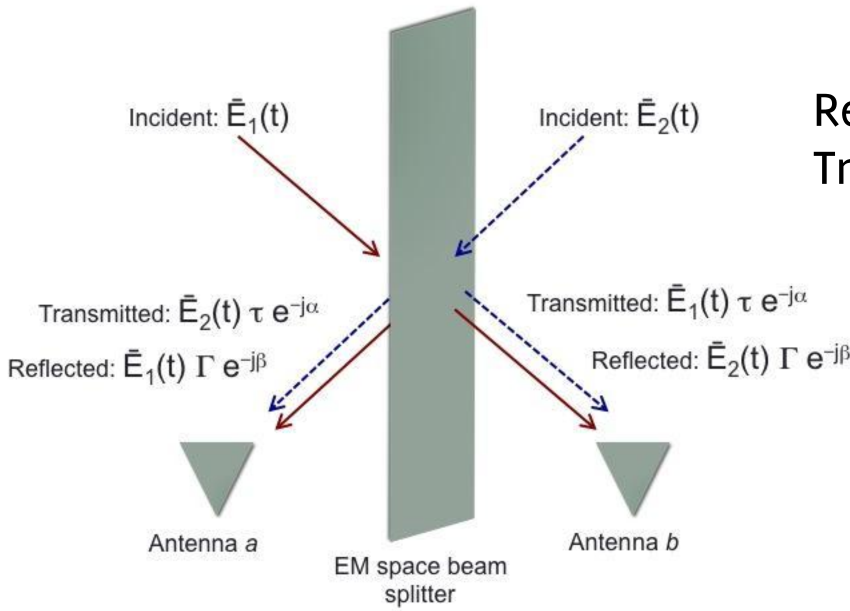


Each Antenna receives the same radiation simultaneously

In effect, ensuring the zero baseline.

This concept would be the;
Zero Baseline Radio Interferometer

Response of ZEBRA



Reflection co-eff: $|\Gamma|\, e^{-j\beta}$

Transmission co-eff: $|\tau|\, e^{-j\alpha}$

$$\overline{E_a(t)} = \overline{E_1(t)}\Gamma e^{-j\beta} + \overline{E_2(t)}\tau e^{-j\alpha},$$

$$\overline{E_b(t)} = \overline{E_2(t)}\Gamma e^{-j\beta} + \overline{E_1(t)}\tau e^{-j\alpha}.$$

$$\begin{aligned}\langle \overline{E_a(t)}\ \overline{E_b(t)}^* \rangle \quad = \quad & \Gamma\tau(|\overline{E_2(t)}|^2 + |\overline{E_1(t)}|^2)\cos(\beta-\alpha) \\ & -j\Gamma\tau(|\overline{E_2(t)}|^2 - |\overline{E_1(t)}|^2) \\ & \sin(\beta-\alpha).\end{aligned}$$

The imaginary part will evaluate to zero for the uniform sky.

Lossy Beam splitter???

Conservation of power requires: $\Gamma^2 + \tau^2 = 1$

No time varying magnetic field => curl E = dB/dt = 0

Therefore, balancing of electric fields on either sides beam splitter gives:

$$1 + \Gamma \cdot e^{-j\beta} = \tau \cdot e^{-j\alpha}.$$

Solves to give, $(\beta - \alpha) = \pi/2$

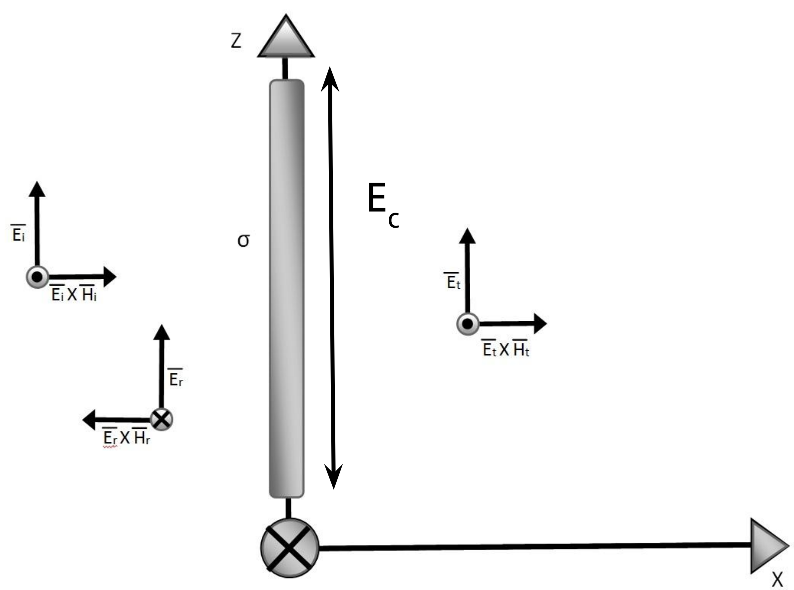
Lossless beam splitter will have zero response to the uniform sky

$$\begin{aligned} \langle \overline{E_a(t)}\ \overline{E_b(t)}^* \rangle \quad = \quad & \Gamma\tau(|\overline{E_2(t)}|^2 + |\overline{E_1(t)}|^2)\cos(\beta - \alpha \\ & -j\Gamma\tau(|\overline{E_2(t)}|^2 - |\overline{E_1(t)}|^2) \\ & \sin(\beta - \alpha). \end{aligned}$$

EM analysis of the Beam Splitter

Case of Normal Incidence

- Splitter is in the yz-plane
- Incoming radiation along the positive x direction



<u>Boundary conditions for the conductive sheet:</u>

Discontinuity in the normal component of E α surface charge density

•No normal E component in case of normal incidence of the incoming wave

Normal component of B is continuous

•No normal B component in case of normal incidence of the incoming wave

Tangential component of E is continuous

$$E_i + E_r - E_t = 0.$$

Discontinuity in tangential component of B α Surface current density

$$H_i - H_r - H_t = \sigma \cdot E_c \cdot \delta x,$$

Selection of the conductance value

$$\Gamma = \frac{S\frac{\eta_o}{2}}{(1+S\frac{\eta_o}{2})}\angle 180^\circ.$$

$$\tau = \frac{1}{1+S\frac{\eta_o}{2}}\angle 0^\circ.$$

Correlator Output

$$\langle \overline{E_a(t)}\ \overline{E_b(t)}^* \rangle = \Gamma\tau(|\overline{E_2(t)}|^2 + |\overline{E_1(t)}|^2)\cos(\beta-\alpha) \\ -j\Gamma\tau(|\overline{E_2(t)}|^2 - |\overline{E_1(t)}|^2) \\ \sin(\beta-\alpha).$$

To maximize the correlator output, the conductance should be a real value, i.e, resistive only

The value of S can be calculated by maximizing Γτ

$$|\Gamma|\cdot|\tau| = \frac{S\frac{\eta_o}{2}}{(1+S\frac{\eta_o}{2})^2},$$

It gives S = 2/377 ⇨ Γ = τ = 0.5 (for normal incidence)

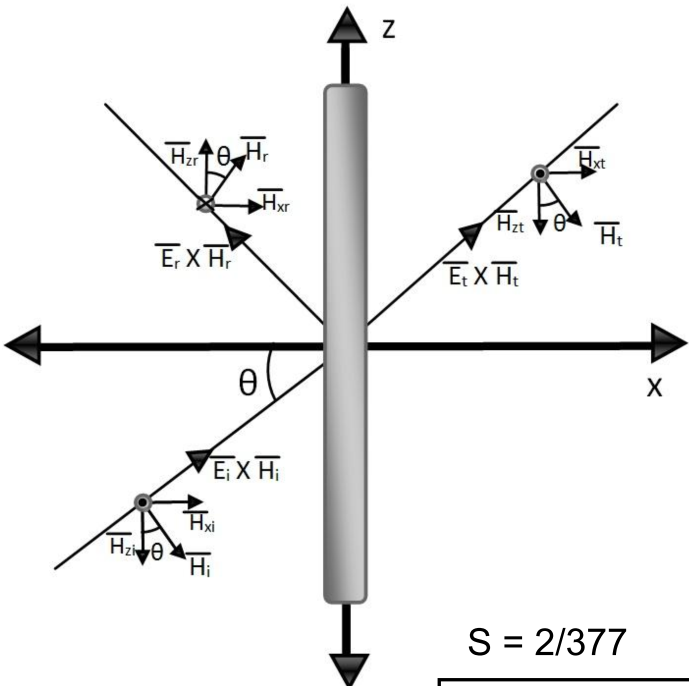
$$\text{Reflected power} = \frac{E_r^2}{\eta_o} = \frac{1}{\eta_o}\left(S\frac{\eta_o}{2}\right)^2\left(\frac{E_i}{1+S\eta_o/2}\right)^2 = \frac{1}{4}\cdot\frac{E_i^2}{\eta_o}.$$

$$\text{Transmitted power} = \frac{E_t^2}{\eta_o} = \frac{1}{\eta_o}\left(\frac{1}{1+S\frac{\eta_o}{2}}\right)^2 = \frac{1}{4}\cdot\frac{E_i^2}{\eta_o}.$$

EM Analysis of the Beam Splitter

Case of Oblique Incidence

H-Plane



S = 2/377

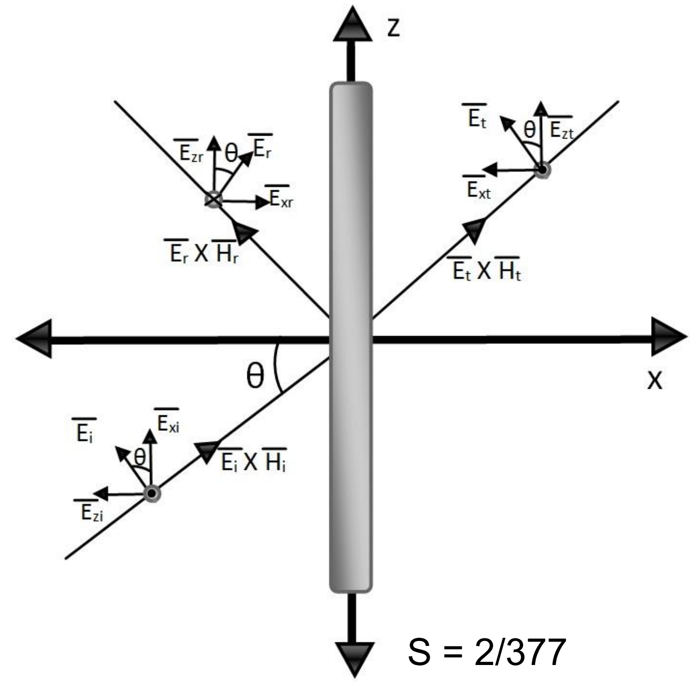
$$\Gamma = \frac{\eta_o S}{2\cos\theta + \eta_o S}\angle 180^\circ$$

$$\tau = \frac{\cos\theta}{\cos\theta + \eta_o S/2}\angle 0^\circ$$

$$\Gamma = \frac{1}{\cos\theta + 1}\angle 180^\circ,$$

$$\tau = \frac{\cos\theta}{\cos\theta + 1}\angle 0^\circ.$$

E-Plane



S = 2/377

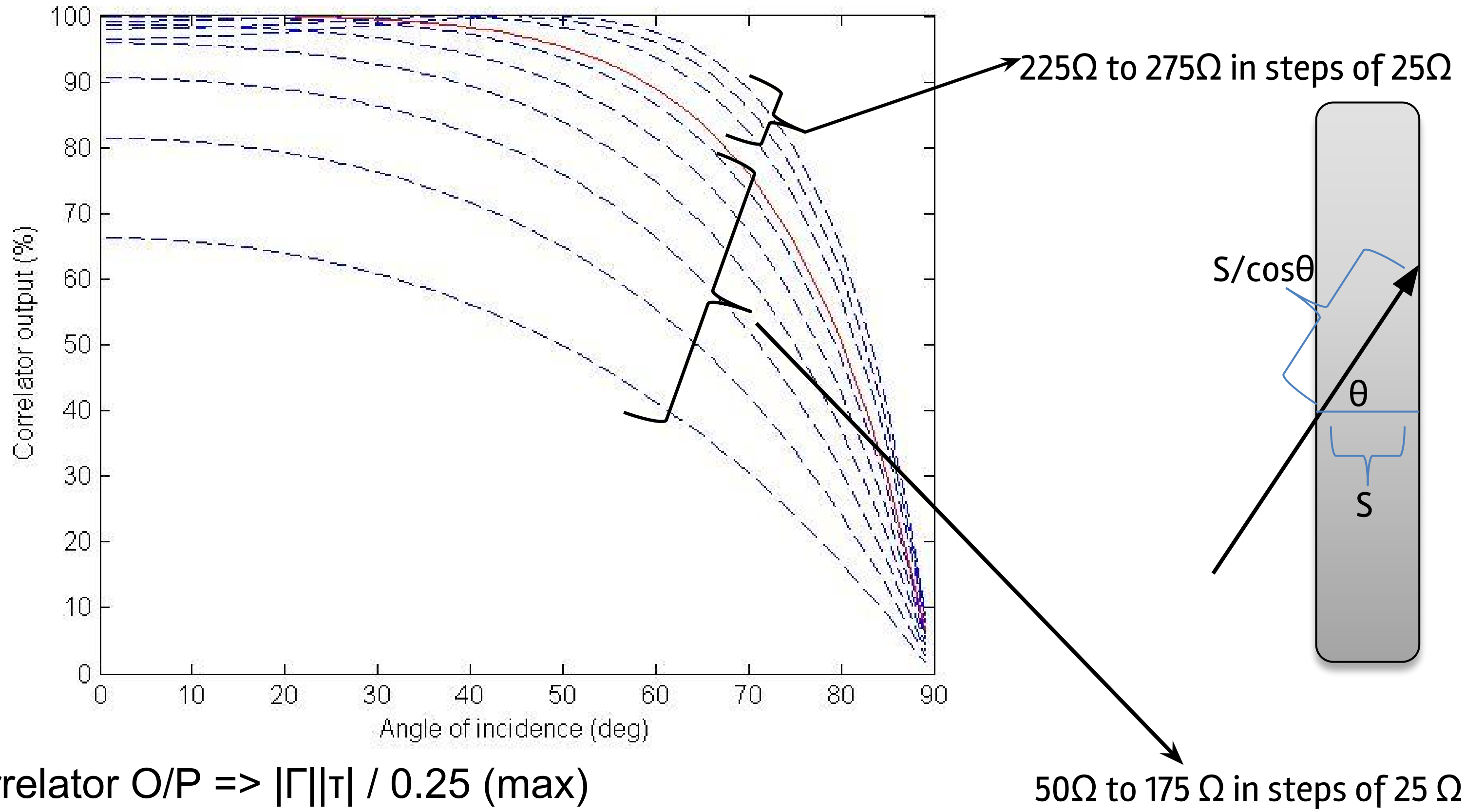
$$\Gamma = \frac{\frac{\eta_o S}{2}\cos\theta}{1 + \frac{\eta_o S}{2}\cos\theta}\angle 180^\circ$$

$$\tau = \frac{1}{1 + \frac{\eta_o S}{2}\cos\theta}\angle 0^\circ$$

$$\Gamma = \frac{\cos\theta}{1 + \cos\theta}\angle 180^\circ$$

$$\tau = \frac{1}{1 + \cos\theta}\angle 0^\circ.$$

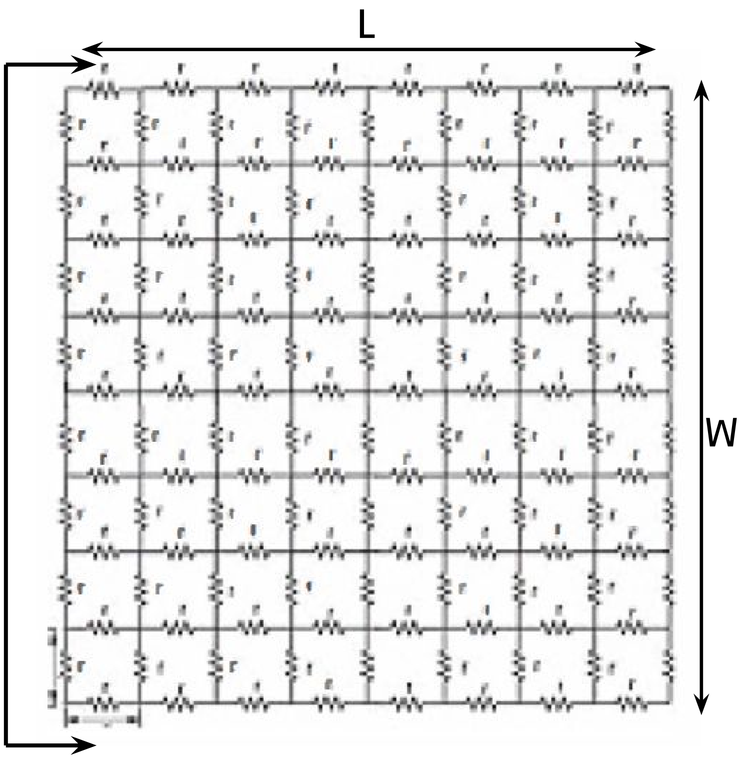
Correlator response Vs Angle of Incidence

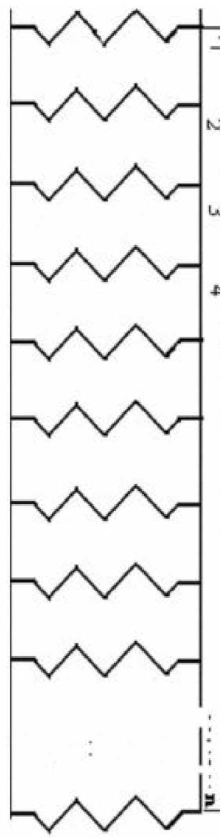


Correlator O/P => |Γ||τ| / 0.25 (max)

- Normal incidence, $\eta_o/2$ gives 100% output
- Effective conductance increases with incidence angle
- Higher resistance gives maximum response at some angle where the effective S evaluates to 2/377

Resistive Square Grid for a Resistive Sheet





Let a = length of each square grid,

r = resistor value of each chip resistor,

n = total number of resistors in one row,

= L/a = W/a

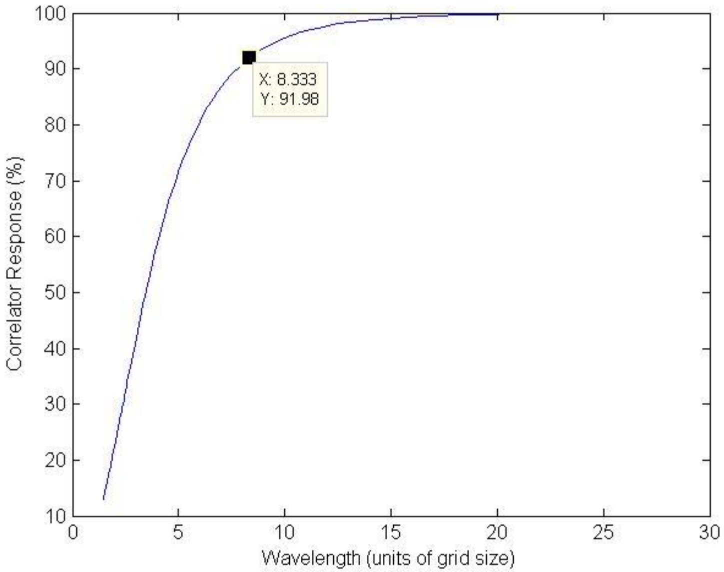
$$R = \frac{(L/a)*r}{W/a}$$

= r (L/W is one for a square grid)

The beam splitter is constructed as a square resistive grid network where the resistance value of each resistor in the grid is $\eta_o/2$

Design Considerations

a.) Grid Size



M. I. Astrakhan, Reflecting and Screening Properties of Plane Wire Grids, Radio Engineering, Vol. 23, No. 1, 1968.

The grid size sets the limit on the upper frequency of operation

If the lowest λ is > 8*a, then loss is <10% (~frequency independent)

b.) Skin Depth

The actual conductance the EM wave encounters;

$$S = \sigma \cdot \delta x\{1 - e^{-\delta x/\delta_s}\}$$

For the operation of the beam splitter:

$$\sigma \cdot \delta x\{1 - e^{-\delta x/\delta_s}\} = \eta_o/2$$

If δx >> δs ; ac conductance = $\sigma \cdot \delta_s$

If δx << δs ; ac conductance = $\sigma \cdot \delta x$

For frequency independent operation

$$\sigma \cdot (\delta x)^2 \ll 1/(\pi f \mu_o \mu_r)$$

$$\boxed{\delta x \ll (6.913/f_{100\mathrm{MHz}})\ \mathrm{cm}}$$

Fabrication of a Prototype

A resistive square grid was constructed with grid size of 10cm.

It is $\lambda/8$ for 375MHz.

Strapping tapes were used for support. PP (1.5–2.5) was selected over PET(2.8–4.8).

Each segment has a carbon resistor of value 180Ω (commercially available)

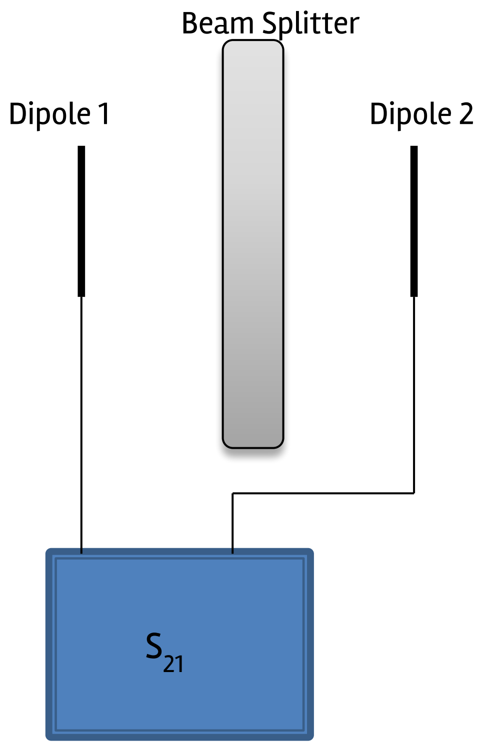
The size was chosen to be 4m X 3m

Testing the Beam Splitter

Frequency range : 50 – 250 MHz (discrete points)

Incidence Modes : Normal, 30º Eplane, 30º Hplane

Transmission Characteristics



Calibration method

Two measurements were made :

with screen (S_{21}^{on})

without screen (S_{21}^{off})

$$\tau_m = \frac{V_{S_{21}^{on}} e^{-j\phi_{S_{21}^{on}}}}{V_{S_{21}^{off}} e^{-j\phi_{S_{21}^{off}}}}.$$

Testing the Beam Splitter

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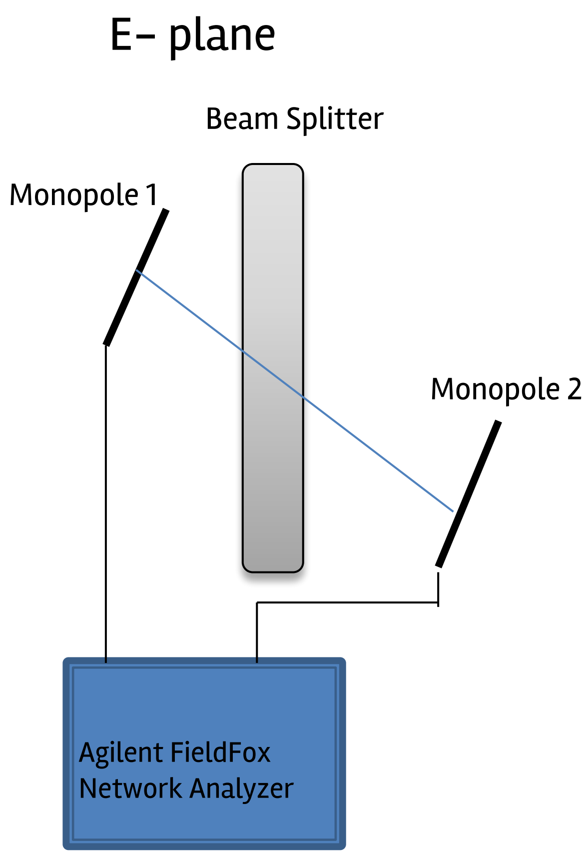
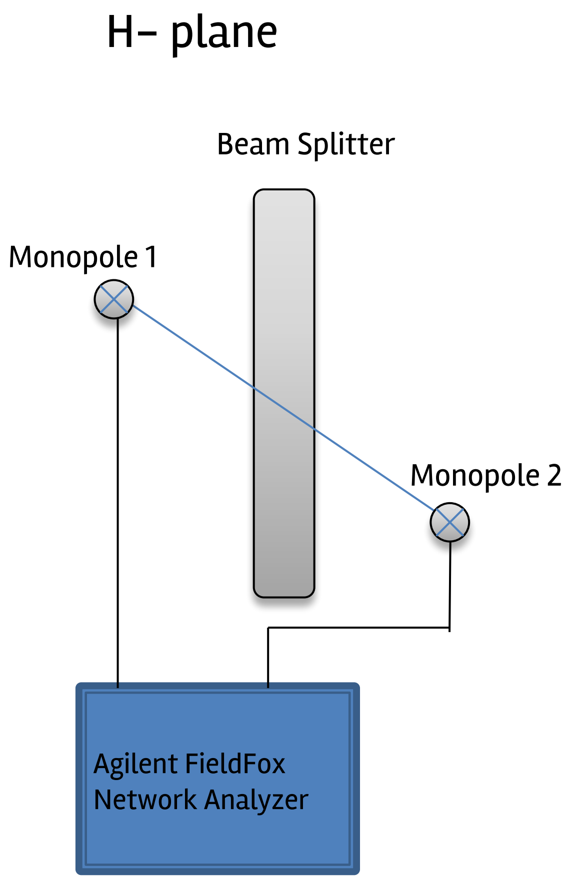
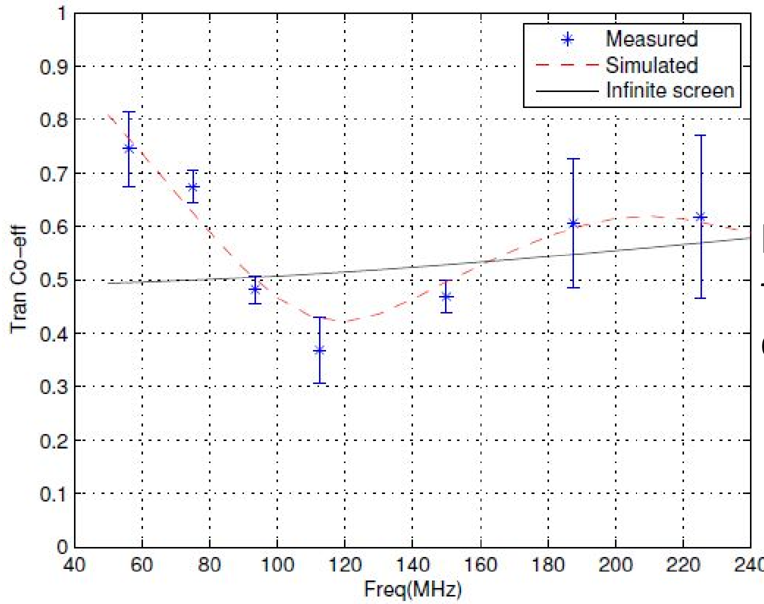
Transmission Characteristics



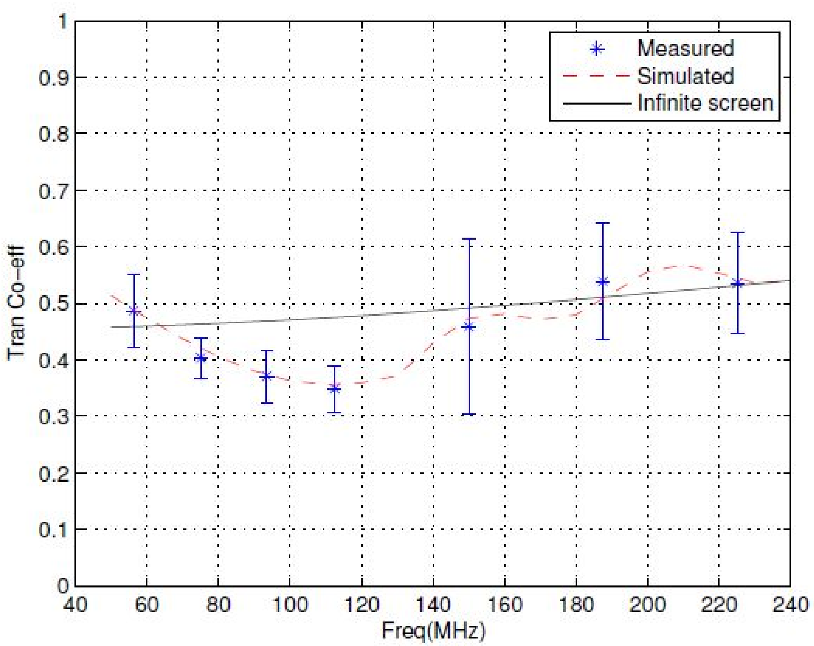


Normal Incidence

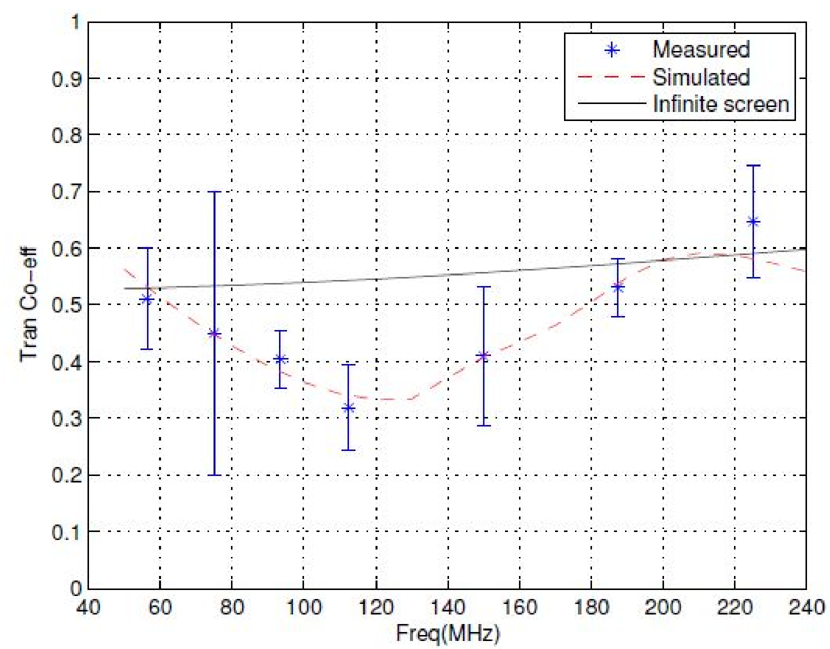


For each frequency the distance between the screen and antenna is adjusted to be 2 X d^2/λ

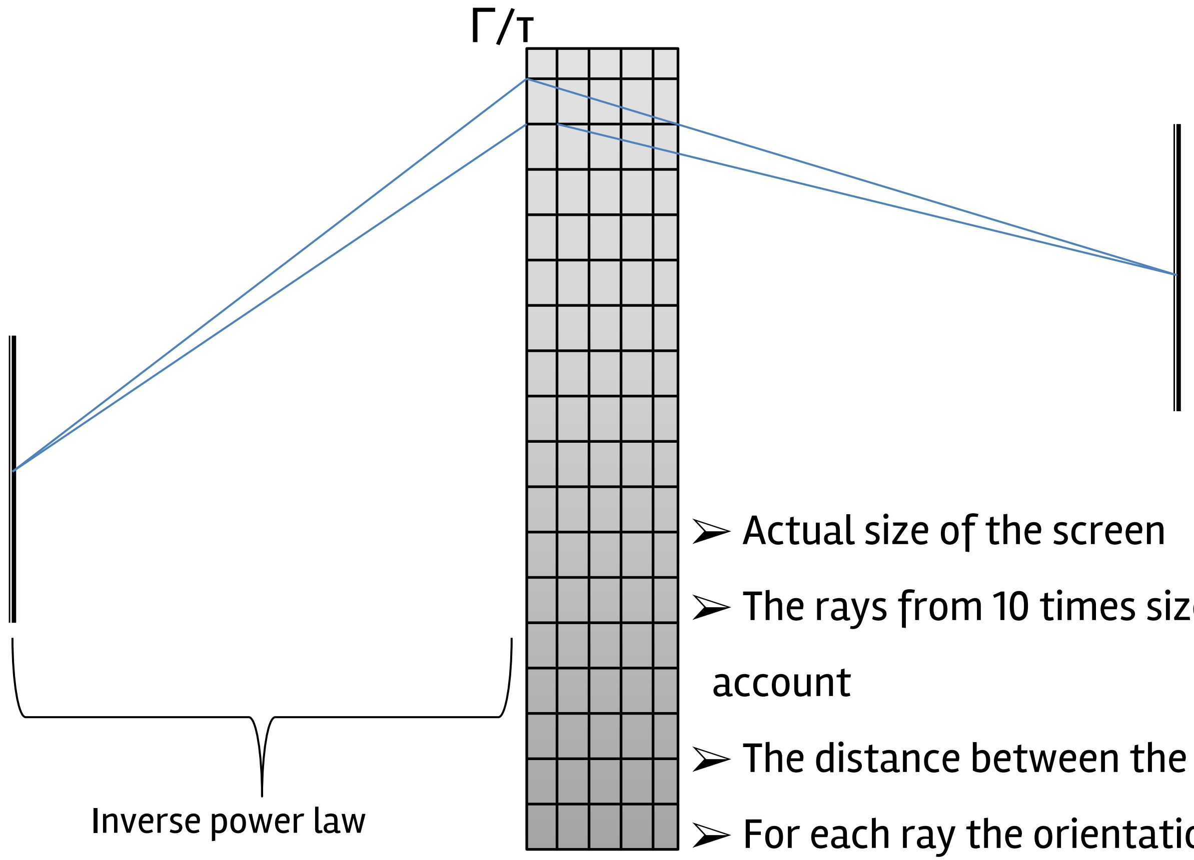
H-Plane



E-Plane



Simulation for Finite Screen



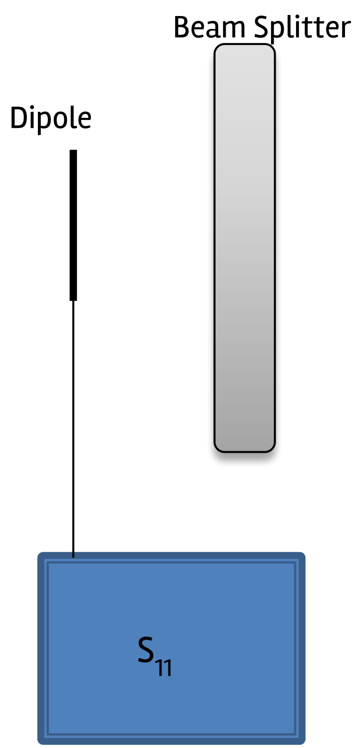
- ➢ Actual size of the screen
- ➢ The rays from 10 times size of the screen were taken into account
- ➢ The distance between the antennas and screen
- ➢ For each ray the orientation of the fields were also taken into account
- ➢ The calculation was made for all the different measurements

Testing the Beam Splitter

Frequency range : 50 – 250 MHz (discrete points)

Incidence Modes : Normal, 30º Eplane, 30º Hplane

Reflection Characteristics



Calibration method

Three measurements were made :

with screen (S_{21}^{on})

without screen (S_{21}^{off})

with reflector (S_{21}^{al})

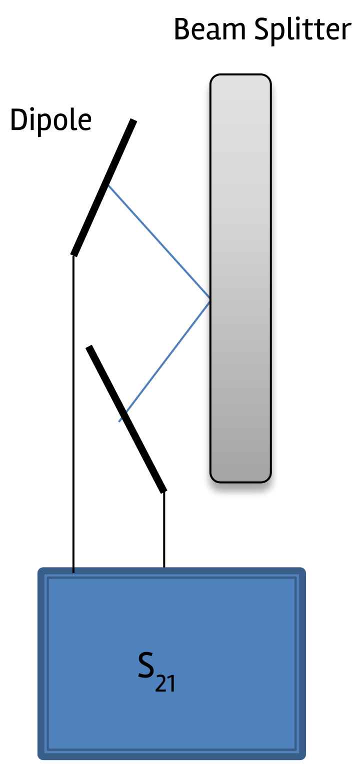
$$\Gamma_m = \frac{V_{S^{\mathrm{on}}_{ij}} e^{-j\phi_{S^{\mathrm{on}}_{ij}}} - V_{S^{\mathrm{off}}_{ij}} e^{-j\phi_{S^{\mathrm{off}}_{ij}}}}{V_{S^{\mathrm{al}}_{ij}} e^{-j\phi_{S^{\mathrm{al}}_{ij}}} - V_{S^{\mathrm{off}}_{ij}} e^{-j\phi_{S^{\mathrm{off}}_{ij}}}}.$$

Testing the Beam Splitter

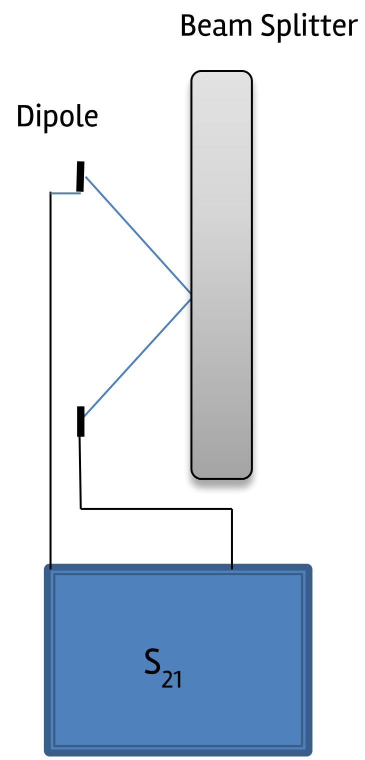
Frequency range : 50 – 250 MHz (discrete points)

Incidence Modes : Normal, 30º Eplane, 30º Hplane

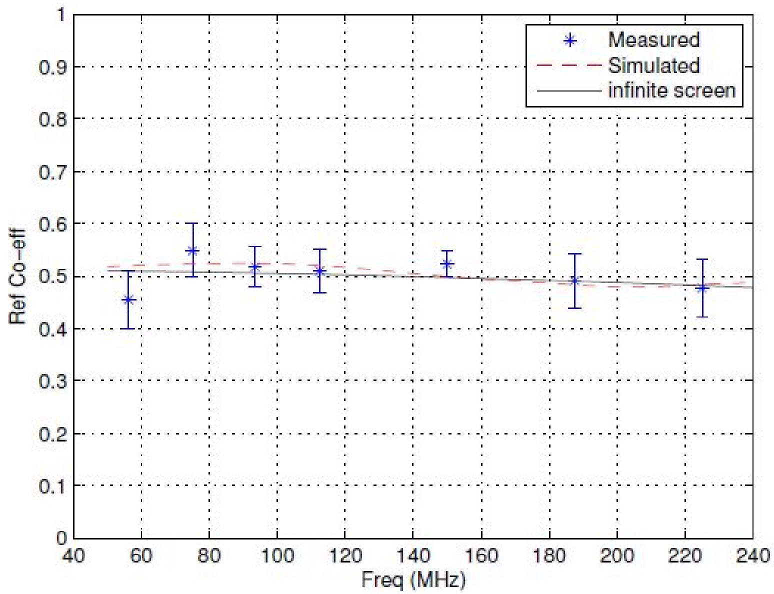
E- plane



H- plane

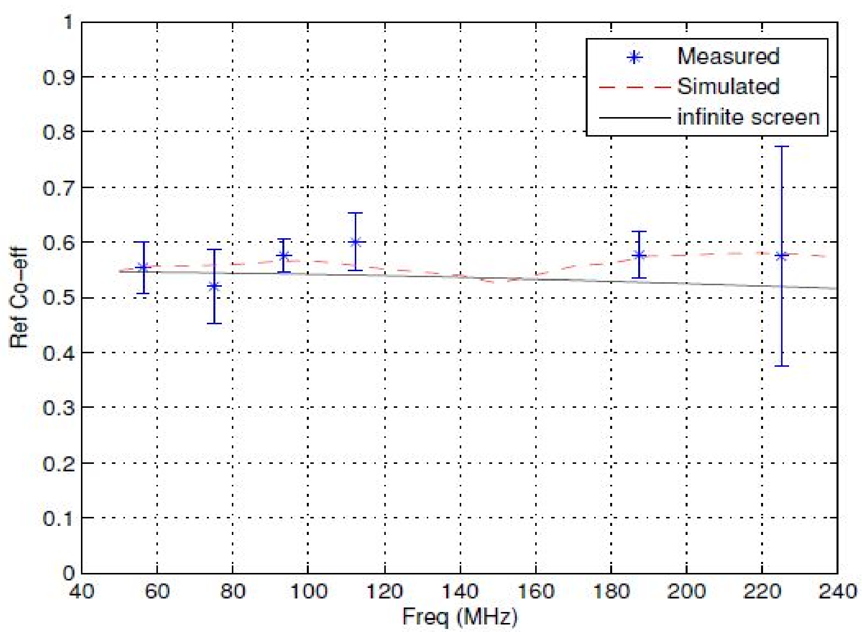


Normal Incidence

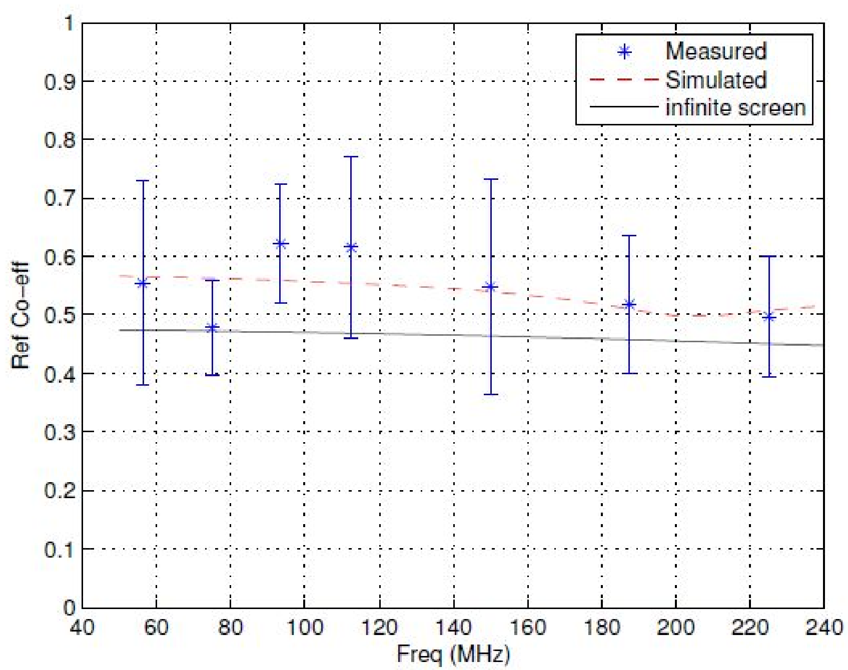


For each frequency the distance between the screen and antenna is adjusted to be 2 X d^2/λ

H–Plane



E–Plane



Summary

- Interferometers are a good way of eliminating the additive receiver noise
- For detection of the common mode power, zero baseline is required
- Beam splitter & 2-element interferometer helps in achieving the effective ZEBRA
- The splitter needs to have a finite conductance (lossy) for ZEBRA to respond to the uniform sky
- The maximum response of the ZEBRA is obtained when the conductance is real and of value 2/377. (Ideally should be frequency independent)
- The beam splitter is realized using a resistive grid. The frequency independent characteristics are observed only at wavelengths at least 8X grid size.
- Designed, constructed and successfully tested the Beam splitter

Next step is to design antennas and step up the Zero Baseline interferometer.

Back up Slides

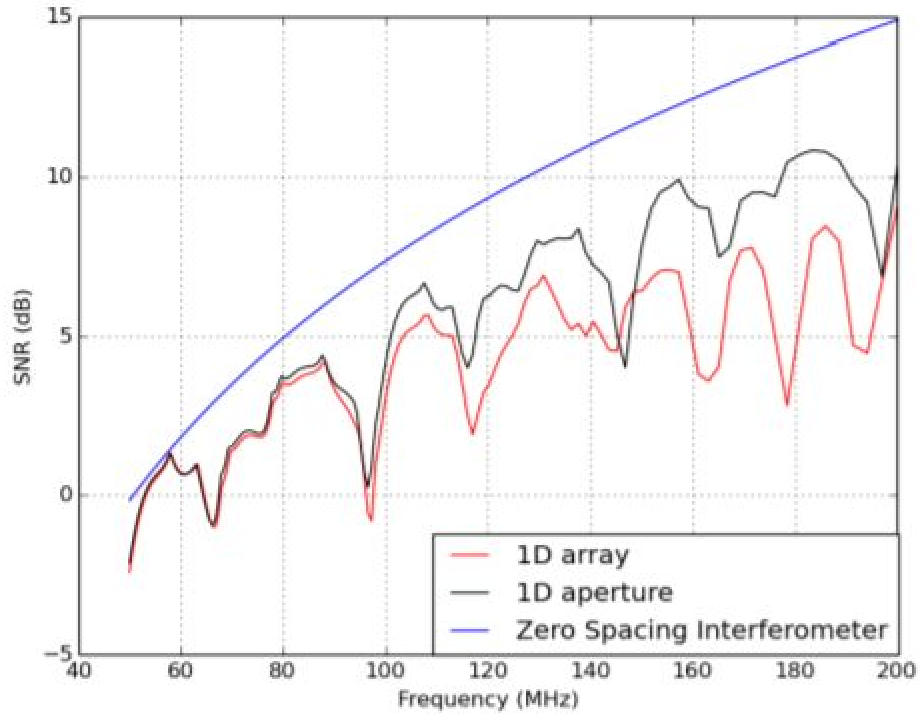
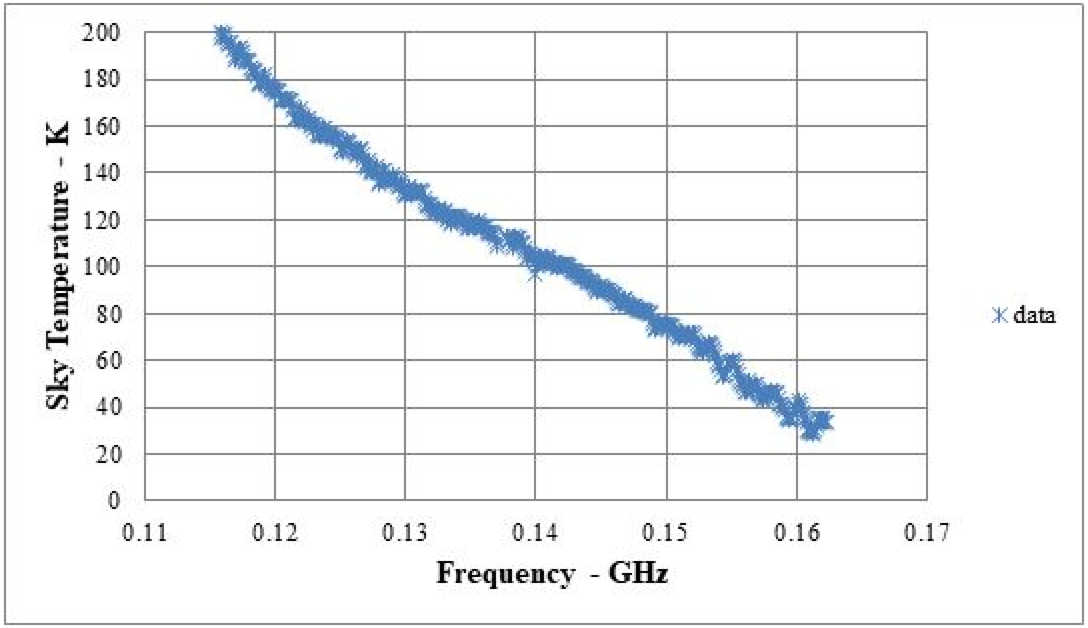
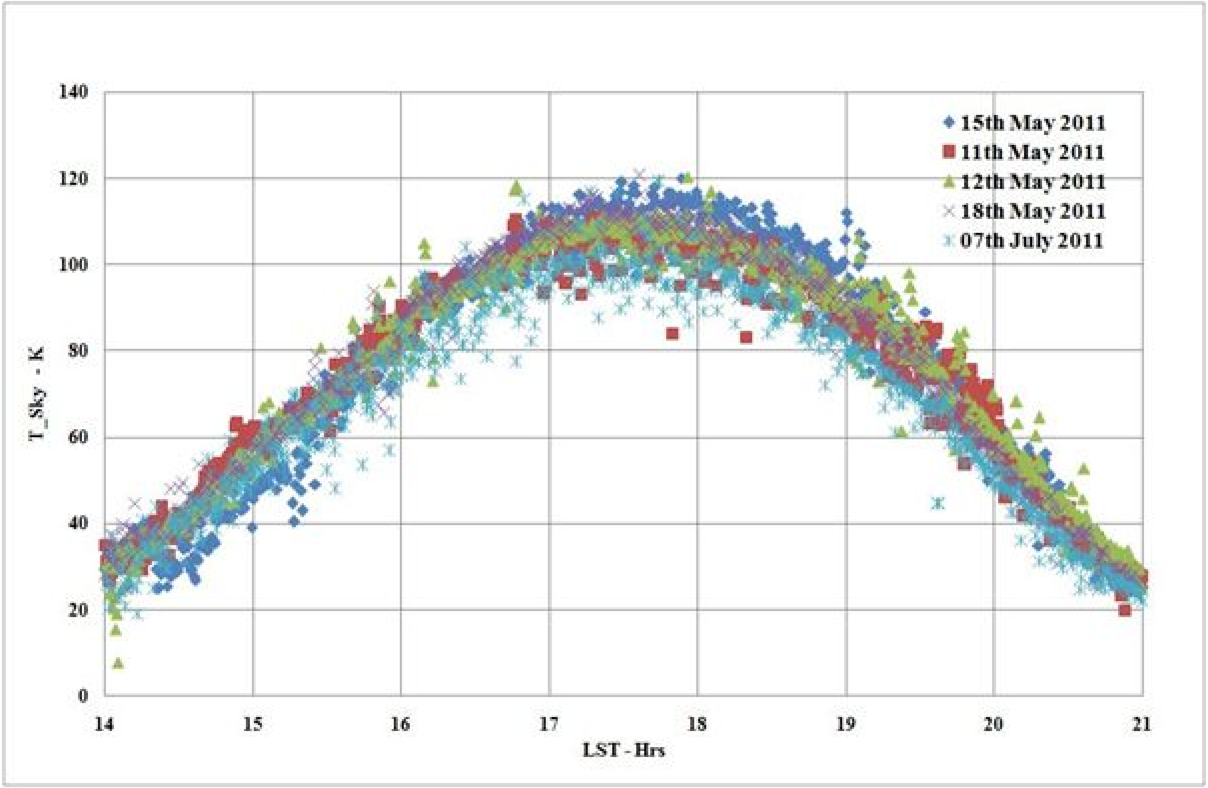


Figure 8. Effective signal-to-noise ratio for the detection of a global signal of amplitude 10 mK. The interferometer array is assumed to consist of three interferometer elements with three baselines formed between the elements; the configuration of the in-line interferometers and 1-D elements are as described in the text. Also shown is the signal-to-noise ratio for a zero-spacing interferometer: a 2-element in-line interferometer of unit dipoles with a resistive sheet in between. 200 hr integration time and 1 MHz spectral bandwidth are assumed.

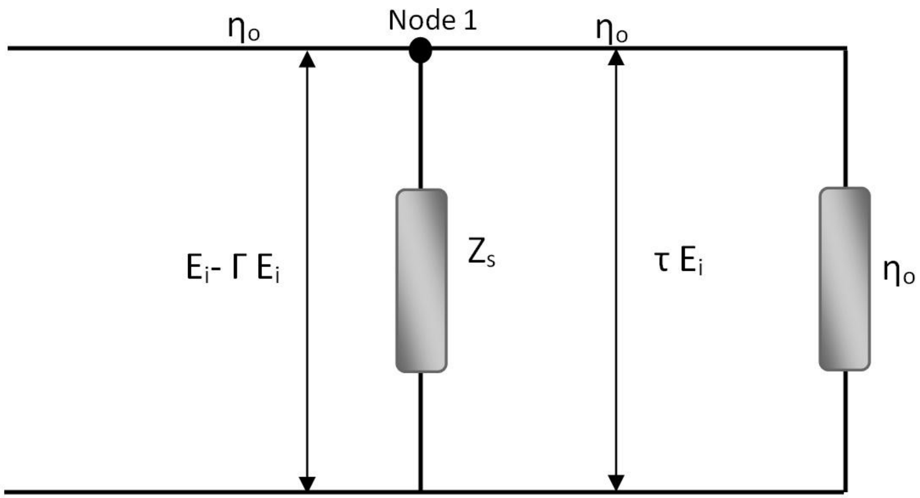
ON THE DETECTION OF GLOBAL 21-CM SIGNAL FROM REIONIZATION USING INTERFEROMETERS
Saurabh Singh , Ravi Subrahmanyan , N. Udaya Shankar , and A. Raghunathan1



Sky Temperature - K
200
180
160
140
120
100
80
60
40
20
0
0.11
0.12
0.13
0.14
0.15
0.16
0.17
Frequency - GHz
data


15th May 2011
11th May 2011
12th May 2011
18th May 2011
07th July 2011
T_Sky - K
LST - Hrs
140
120
100
80
60
40
20
0
14
15
16
17
18
19
20
21

Transmission line analogy analysis



Reflection coefficient @ node 1

$$\Gamma = \frac{-\eta_o/2}{\eta_o/2 + Z_s}$$

Transmission coefficient @ node 1

$$\tau = \Gamma + 1 = \frac{Z_s}{\eta_o/2 + Z_s}.$$

Power absorbed @ Zs;

$$P_{\text{abs}} = 1 - \Gamma^2 - \tau^2 = \frac{Z_s \eta_o}{(\eta_o/2 + Z_s)^2}.$$

Putting $Z_s = \eta_o/2$

$\Rightarrow \Gamma = -0.5, \tau = 0.5$, Pabs = 0.5

- The free space is modeled by transmission line of same impedance value – 377Ω
- The sheet is taken to be of impedance Zs in shunt
- The infinite long transmission line on the right is modeled with a termination of the same 377 Ω
- The incoming radiation is assumed to be incident from the left

Evaluating Γ and τ of the sheet

In this equation, $$H_i - H_r - H_t = \sigma \cdot E_c \cdot \delta x,$$

Substitute, $H = E/\eta_o$ and $E_t = E_i + E_r$

$$E_r = -S\frac{\eta_o}{2}E_c.$$

The net electric field across the sheet Ec is the sum of Ei and Er

$$E_c = E_i - S\frac{\eta_o}{2}E_c.$$

Will give E_r ;

$$E_r = -S\frac{\eta_o}{2}\frac{E_i}{(1+S\eta_o/2)}.$$

$$\Gamma = \frac{S\frac{\eta_o}{2}}{(1+S\frac{\eta_o}{2})}\angle 180^\circ.$$

$$\tau = \frac{1}{1+S\frac{\eta_o}{2}}\angle 0^\circ.$$