Calibrating Radio Arrays without Visibilities Using the E-field Parallel Imaging Calibration (EPICal)

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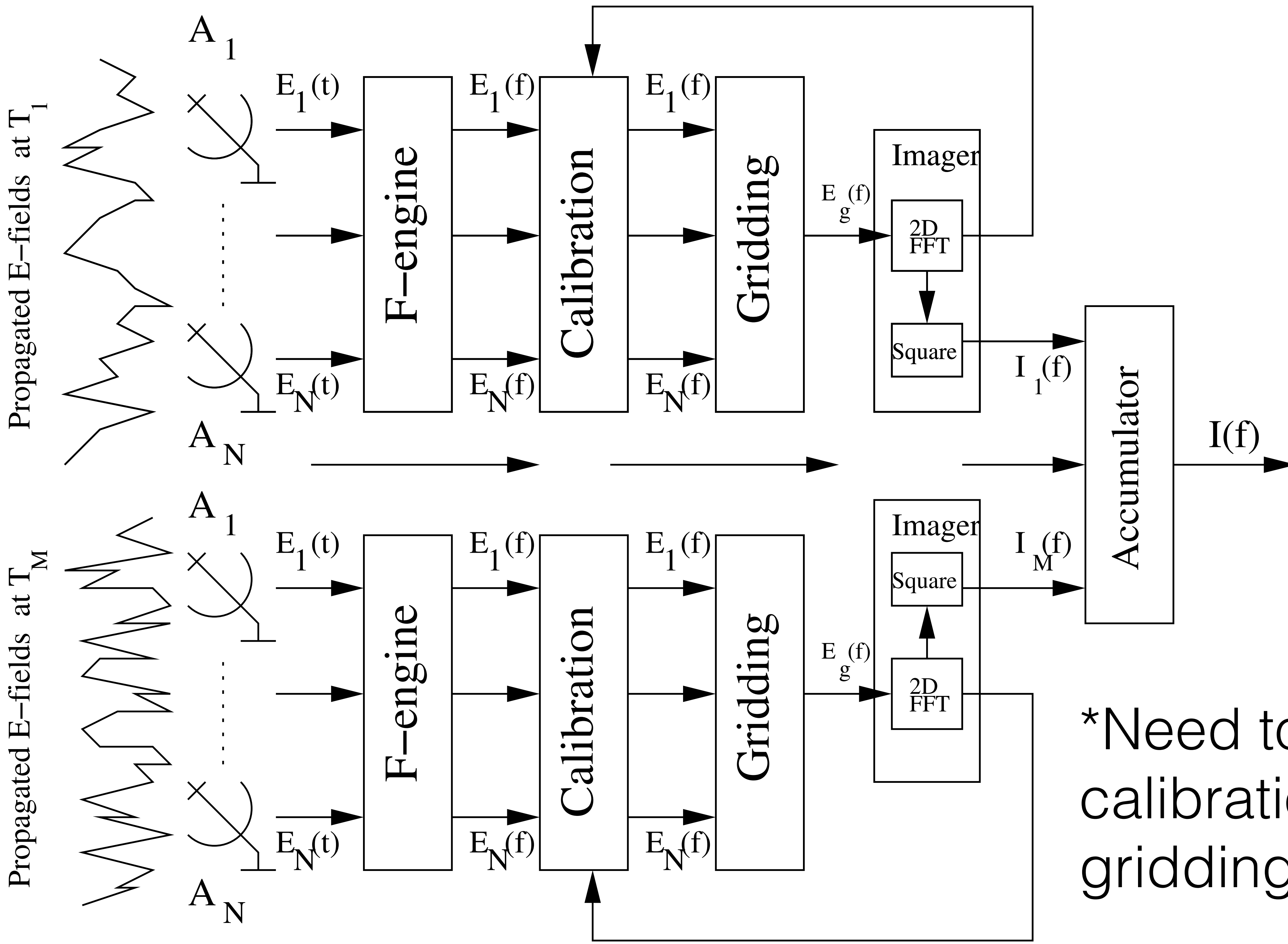
MOFF Data Flow



*Need to apply calibration before gridding.

Calibration Problem

E-field incident on ground related to sky by FT:

$$\widetilde{E}(\mathbf{r}, f, t) = \int E(\hat{\mathbf{s}}, f, t) e^{-2\pi i \mathbf{r}\cdot\hat{\mathbf{s}}}\, d^2\hat{\mathbf{s}}$$

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Antenna integrates with voltage pattern:

$$\widetilde{E}_a^T(f, t) = \int \widetilde{W}_a(\mathbf{r} - \mathbf{r}_a) \widetilde{E}(\mathbf{r}, f, t) \, d^2\mathbf{r}$$

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Further corruption modeled as complex gain and noise

$$\widetilde{E}_a(f, t) = g_a(f, t) \widetilde{E}^T_a(f, t) + n_a(f, t)$$

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Goal is to solve for the gain factor

Calibration Problem

- No visibilities
- MOFF algorithm mixes antenna signals
  - $\Rightarrow$ Must apply calibration at front end
- Scale < $\mathcal{O}(N \log_2 N)$

Simple Case

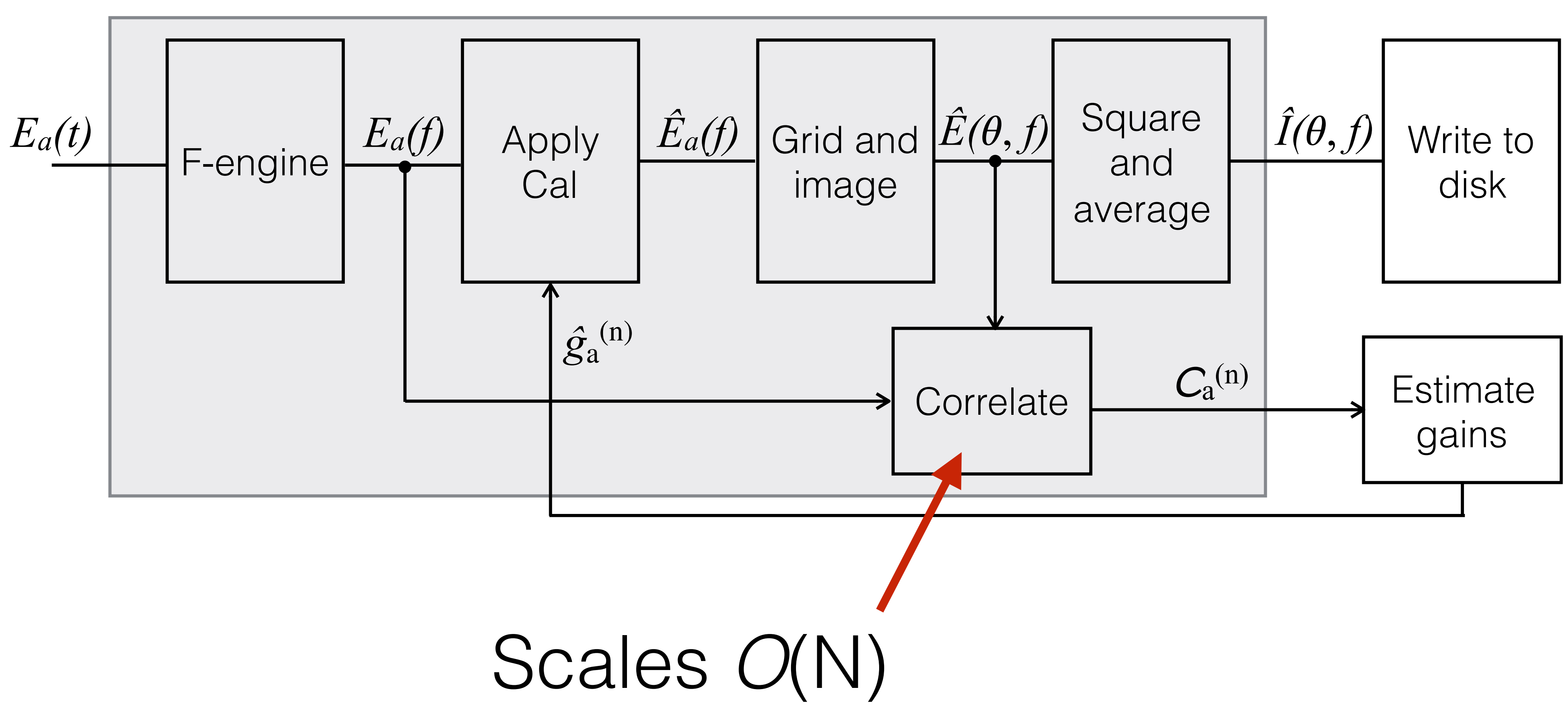
- Single point source, centered on sky
- From Morales, 2011:

$$\sum_b g_b V_{ab} = \left\langle \widetilde{E}_a E'^*(\hat{\mathbf{s}} = 0) \right\rangle_t$$

- Sufficient to determine self-cal solutions (e.g. Mitchell et al. 2008)

Calibration Loop

An iterative approach

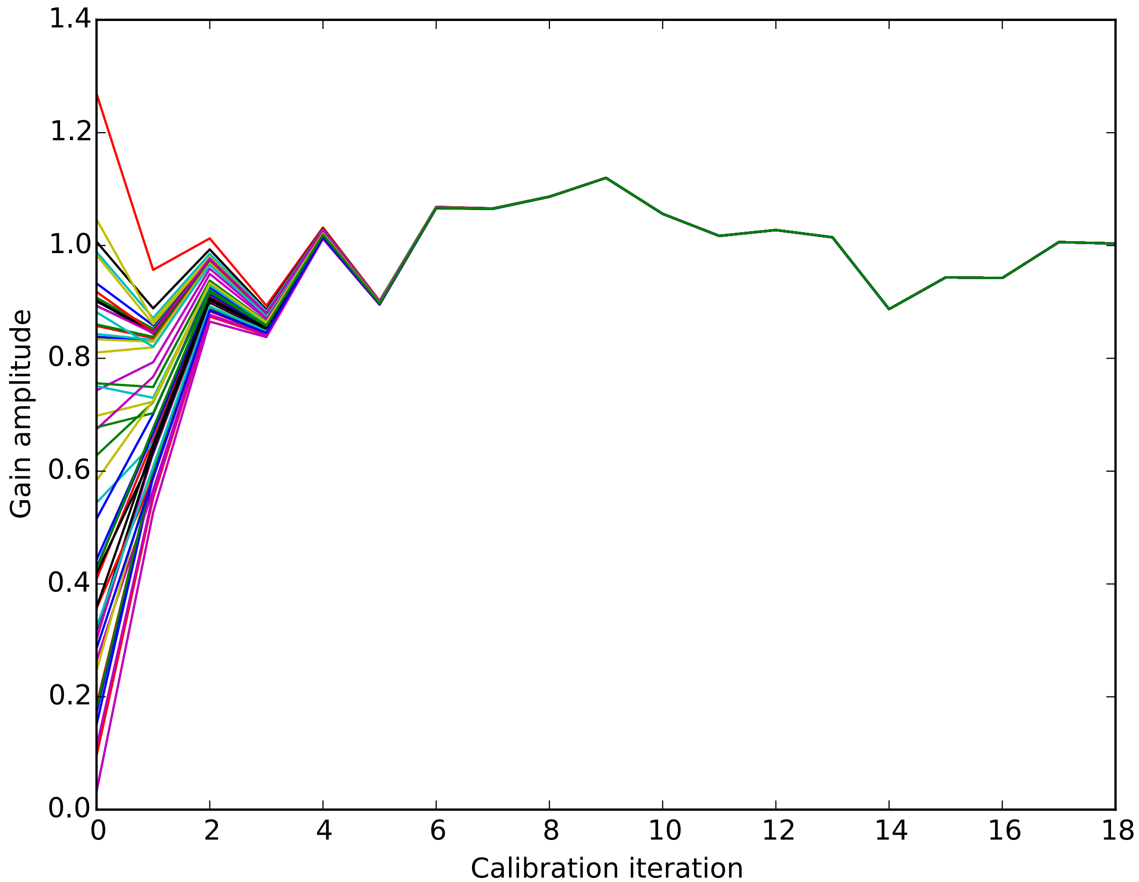


Simple Case

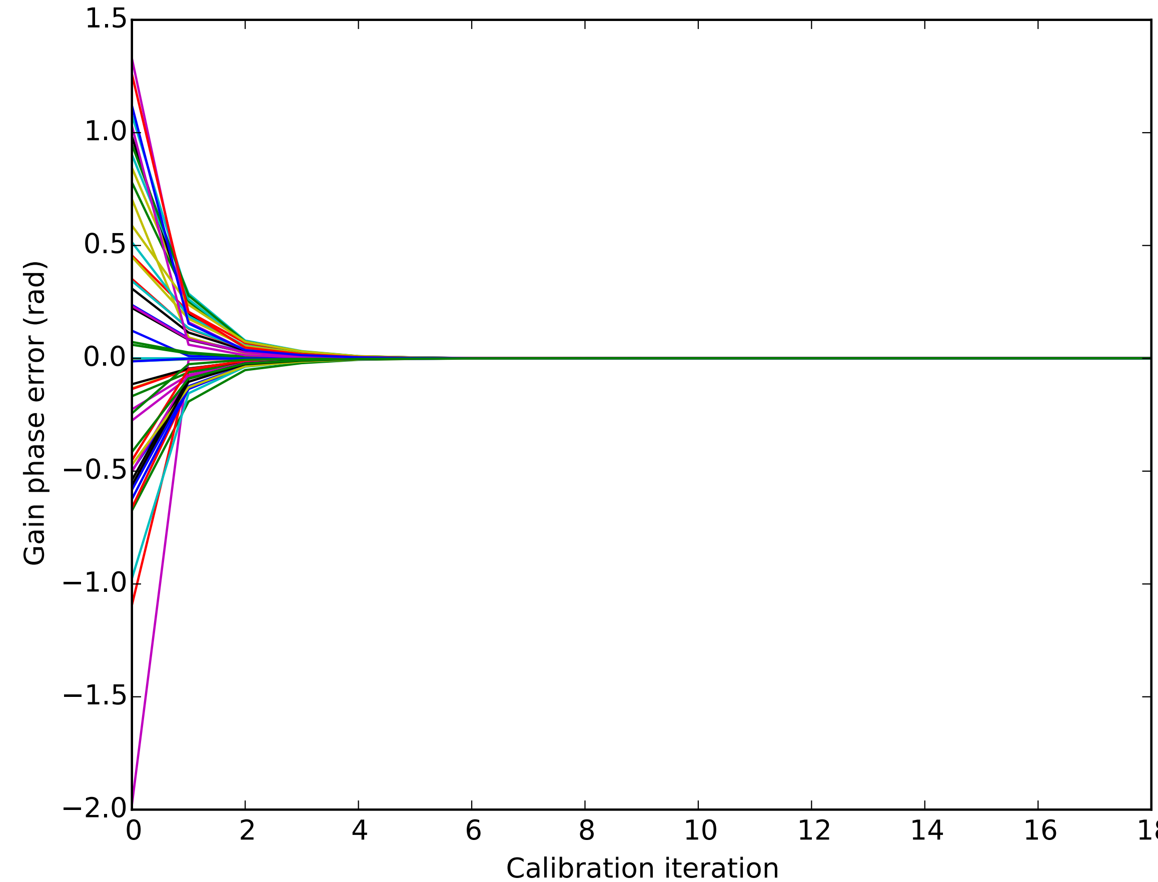
- Simulate using EPIC
  - Use MWA core layout / beam pattern
  - No noise for now
- Start with random “guess” for gains
- Attempt to recover “true” gains

Simple case

Gain Amplitudes

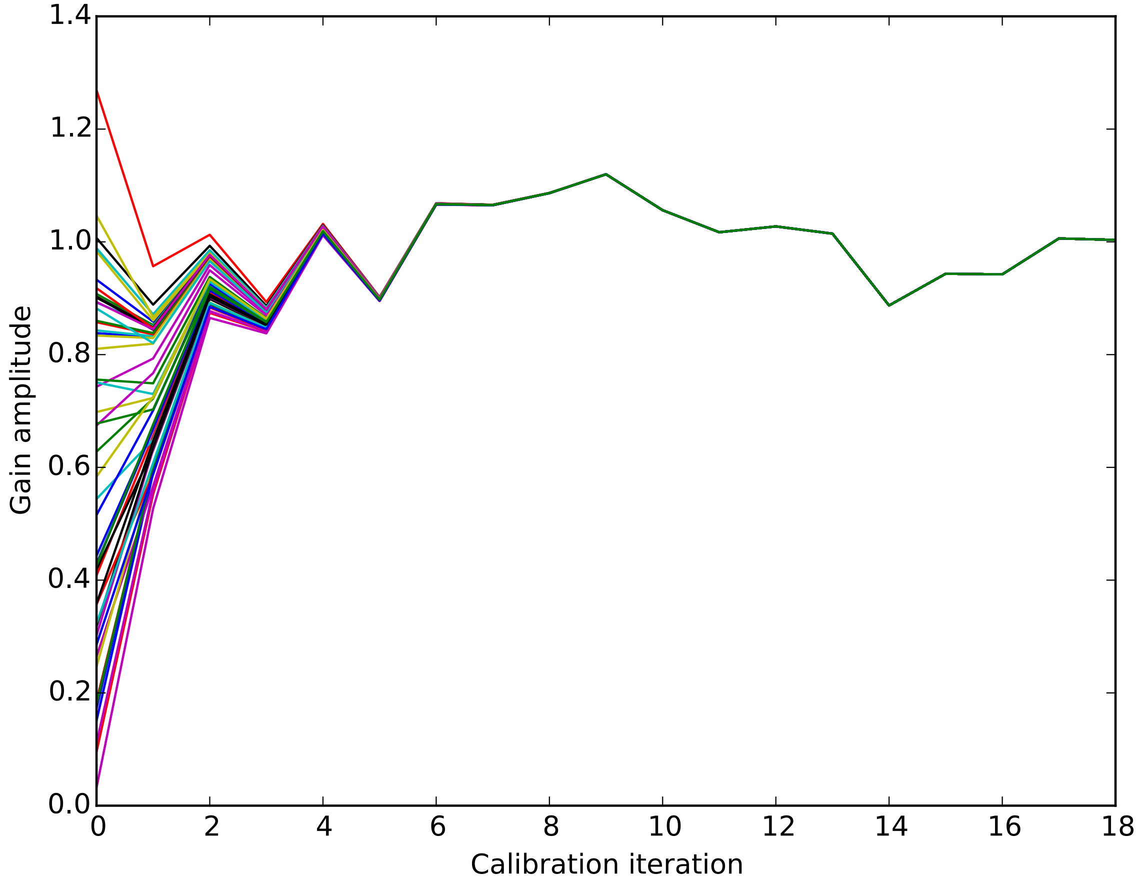


Gain Phase Errors



Simple case

Gain Amplitudes



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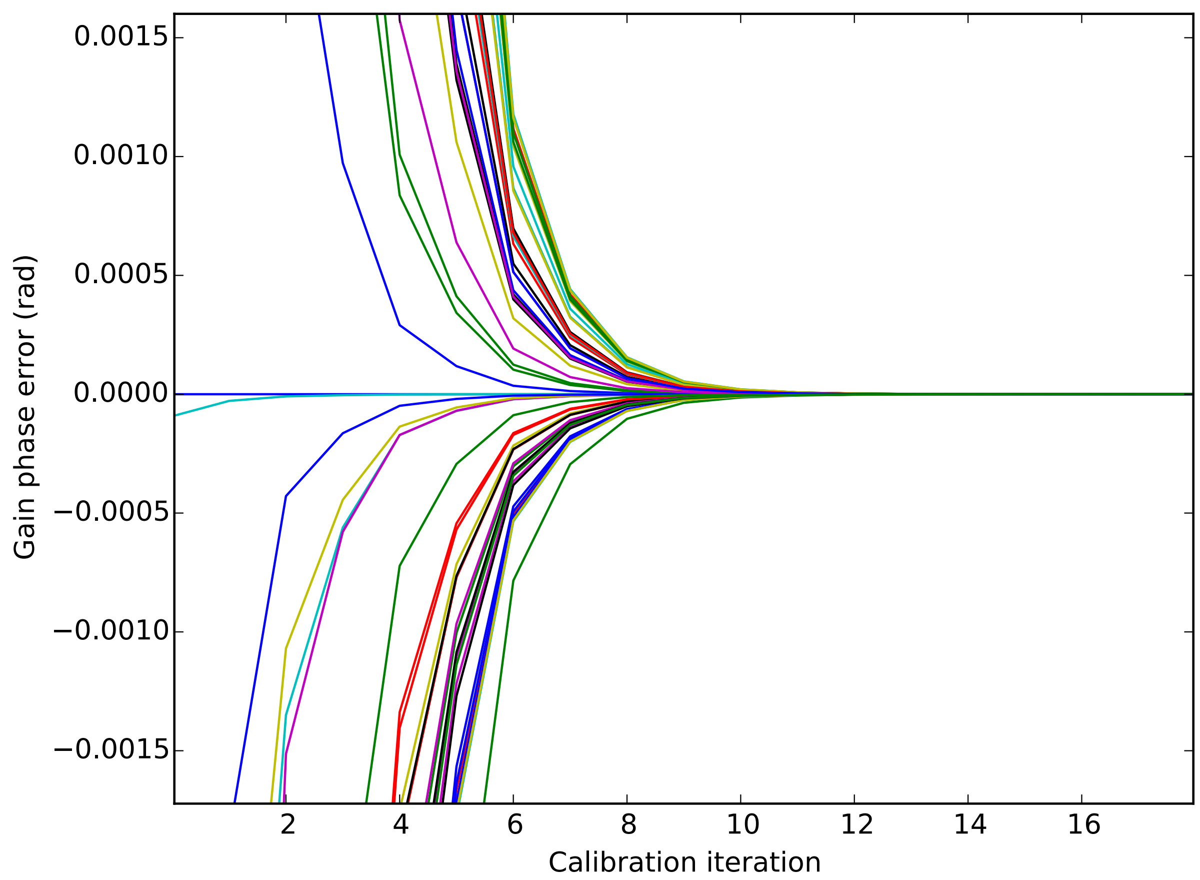


Simple Case

Image Before Calibration

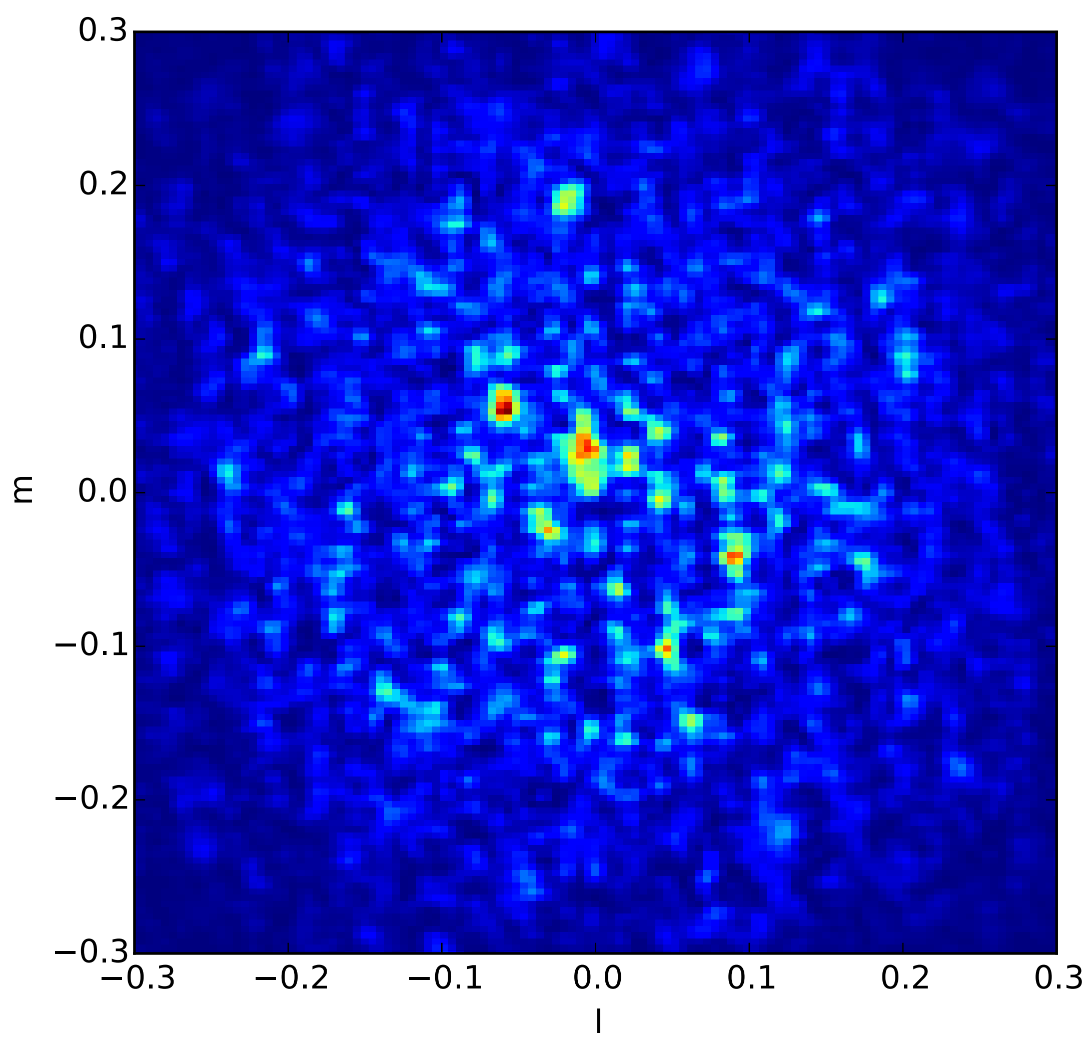
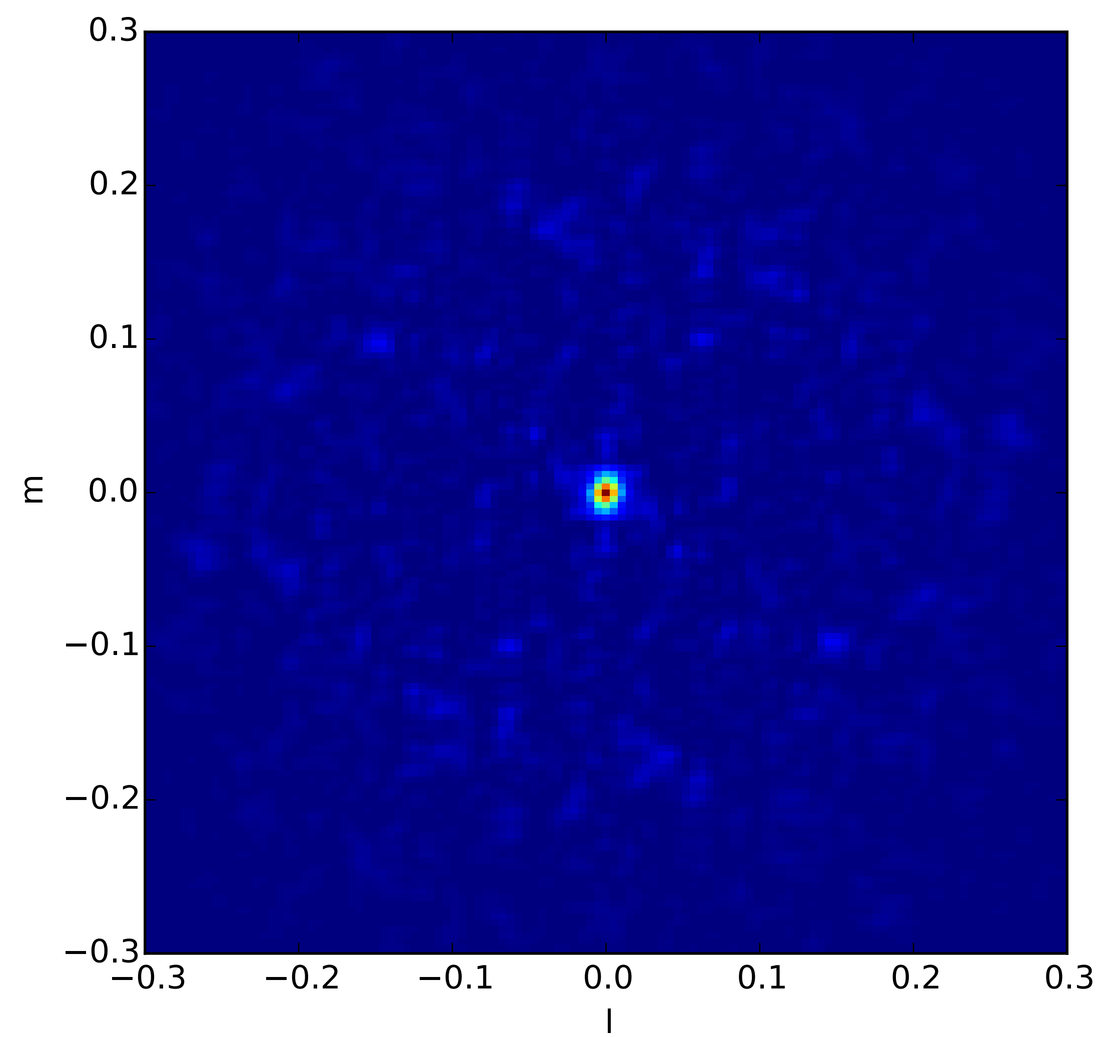


Image After Calibration



Generalize…

$$\mathcal{C}^{(n)}_{a,\hat{\mathbf{s}}_0} \equiv \left\langle \widetilde{E}_a(t) E'^*(\hat{\mathbf{s}}_0, t) \right\rangle_t$$

Inspect this correlation

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$$\ldots$$

$$= \frac{g_a}{N_a} \sum_b \frac{g_b^*}{g_b'^{*(n)}} W_b^*(\hat{\mathbf{s}}_0) e^{-2\pi i \hat{\mathbf{s}}_0 \cdot \mathbf{r}_b} V_{ab}^T$$

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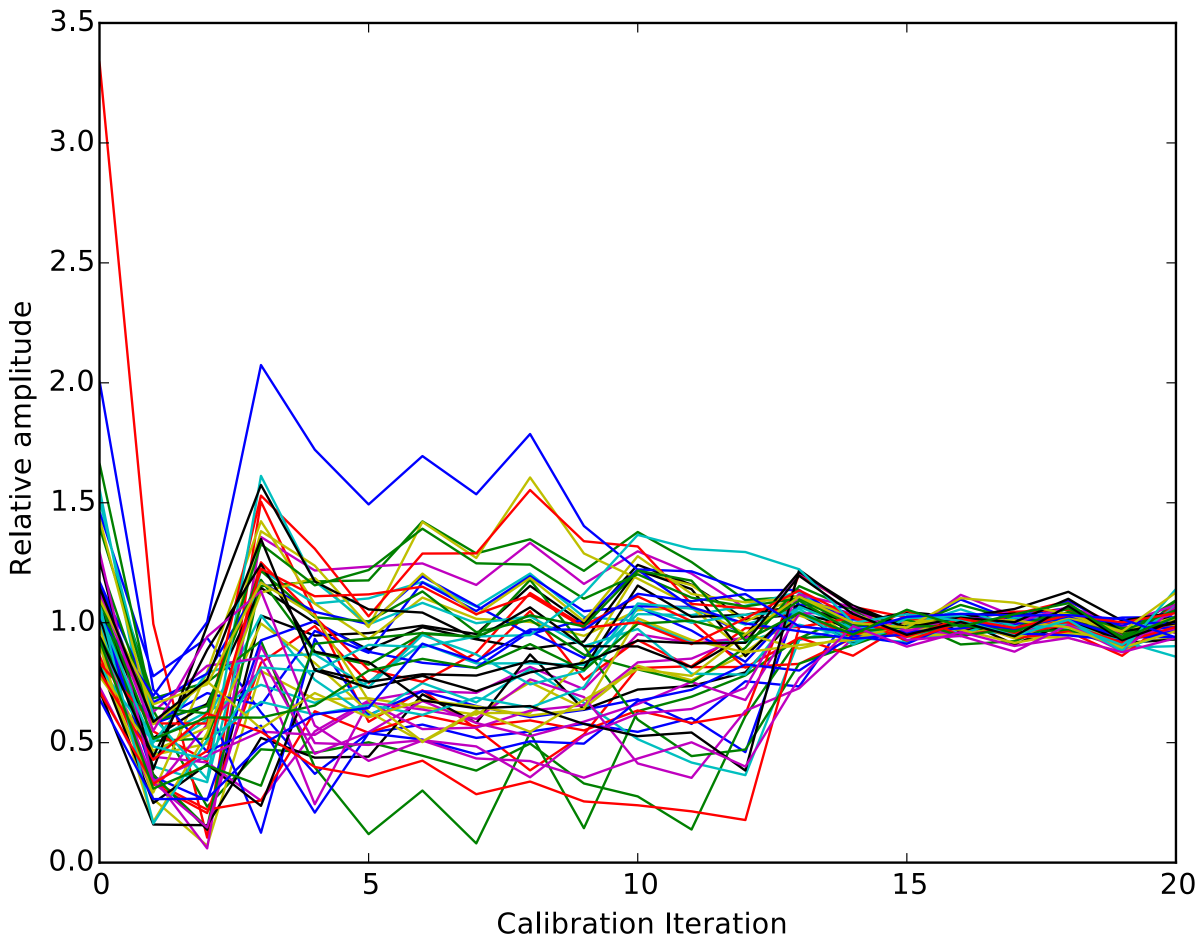
$$\rightarrow g_a'^{(n+1)} = \mathcal{C}_{a,\hat{\mathbf{s}}_0}^{(n)} N_a \left[\sum_b W_b^*(\hat{\mathbf{s}}_0) e^{-2\pi i \hat{\mathbf{s}}_0 \cdot \mathbf{r}_b} V_{ab}^T \right]^{-1}$$

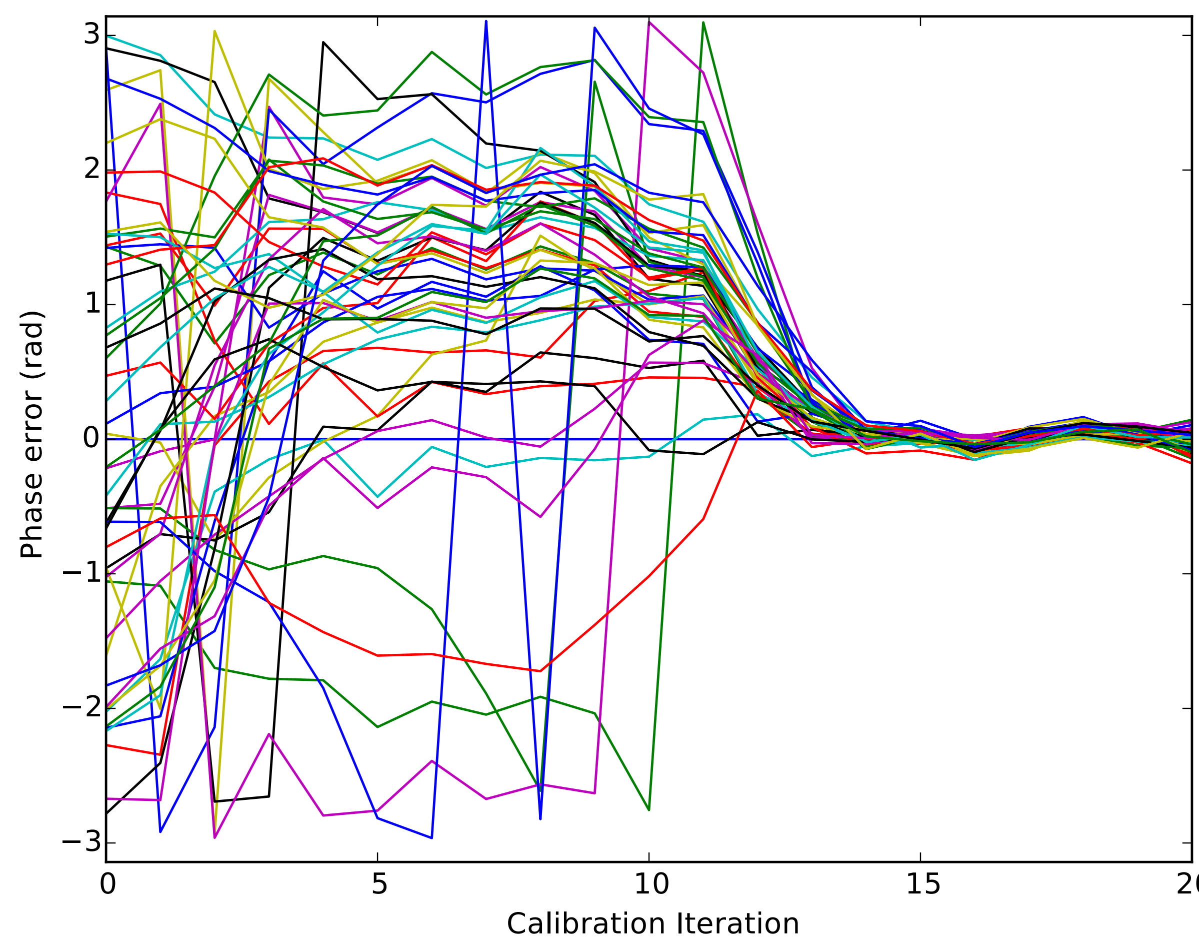
Arbitrary sky, any direction

Simulate more sources

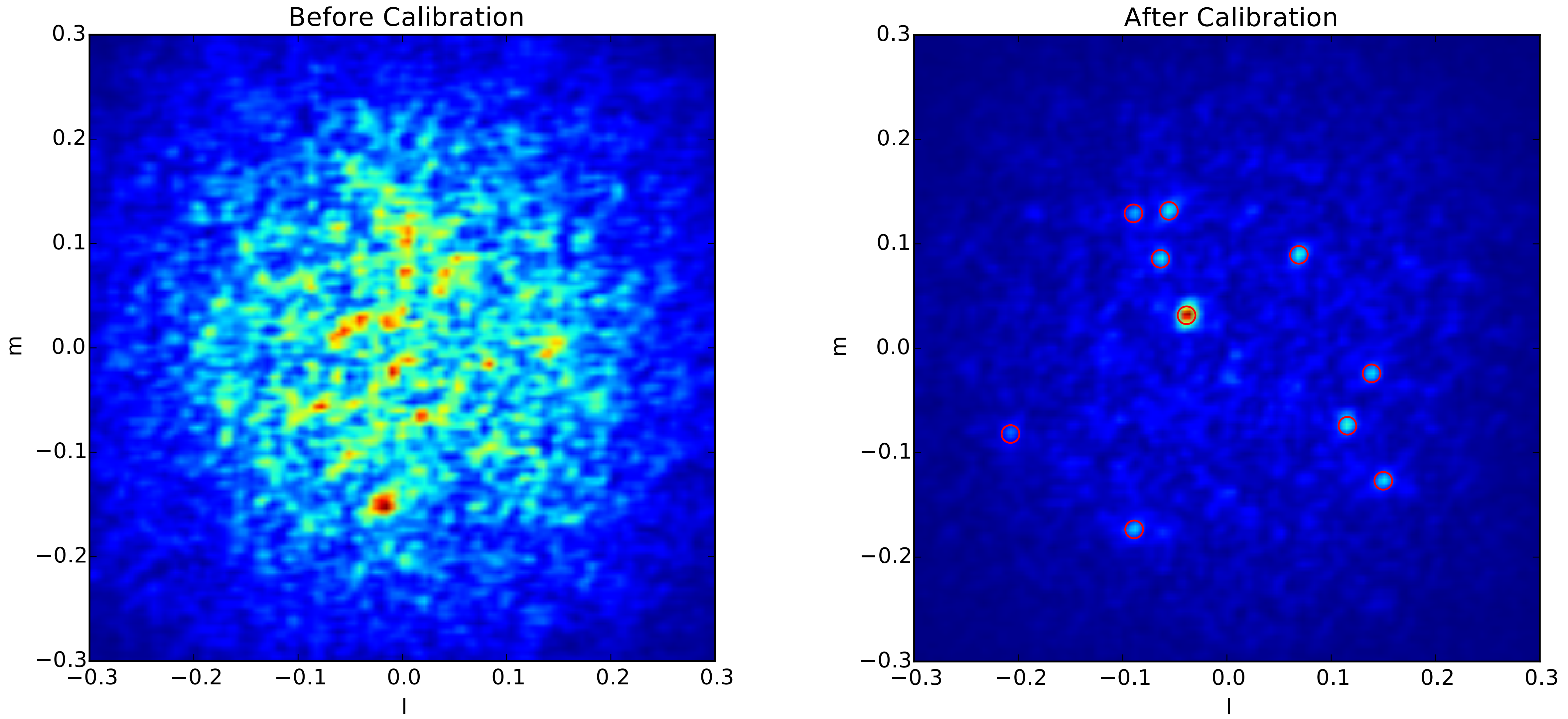
Gain Amplitudes



Gain Phase Errors



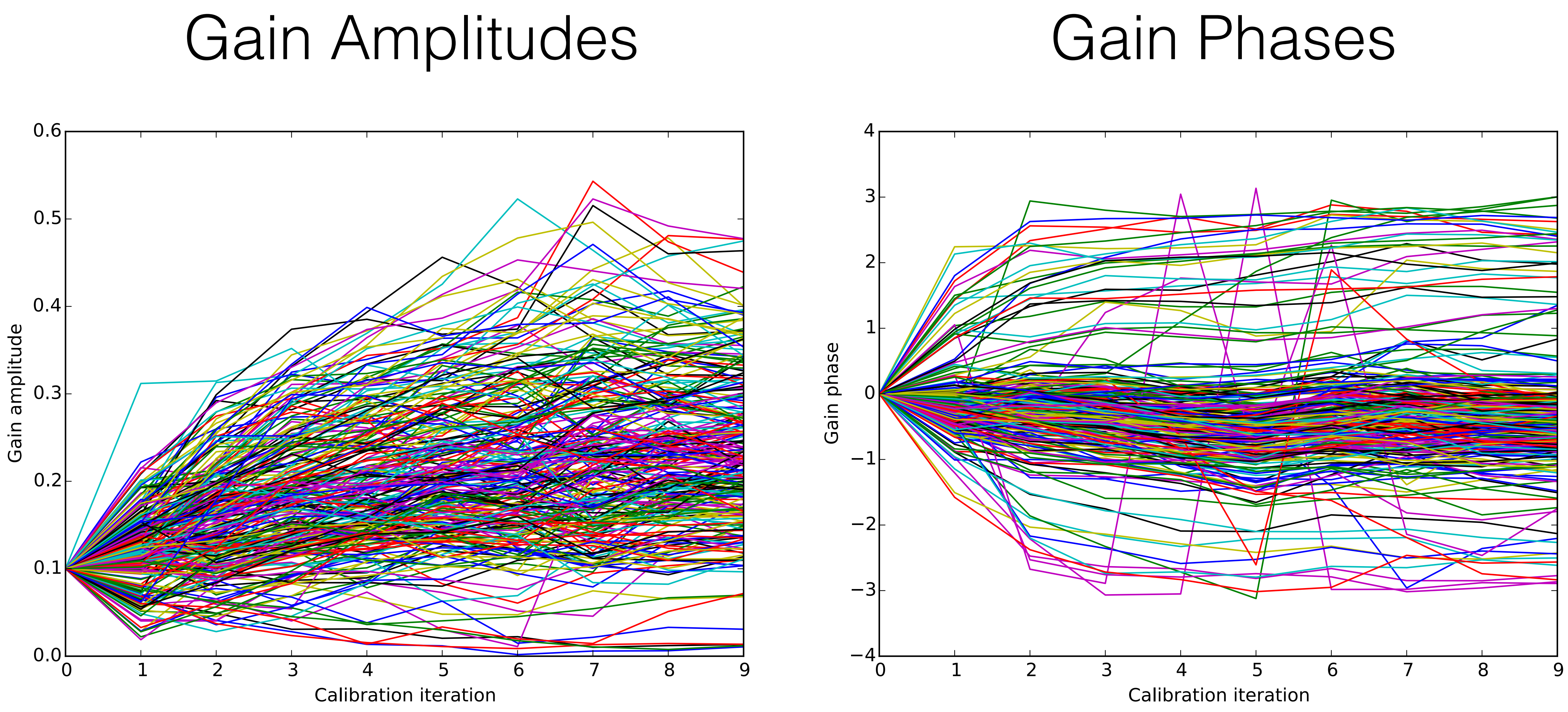
Simulate More Sources



Calibrating LWA data

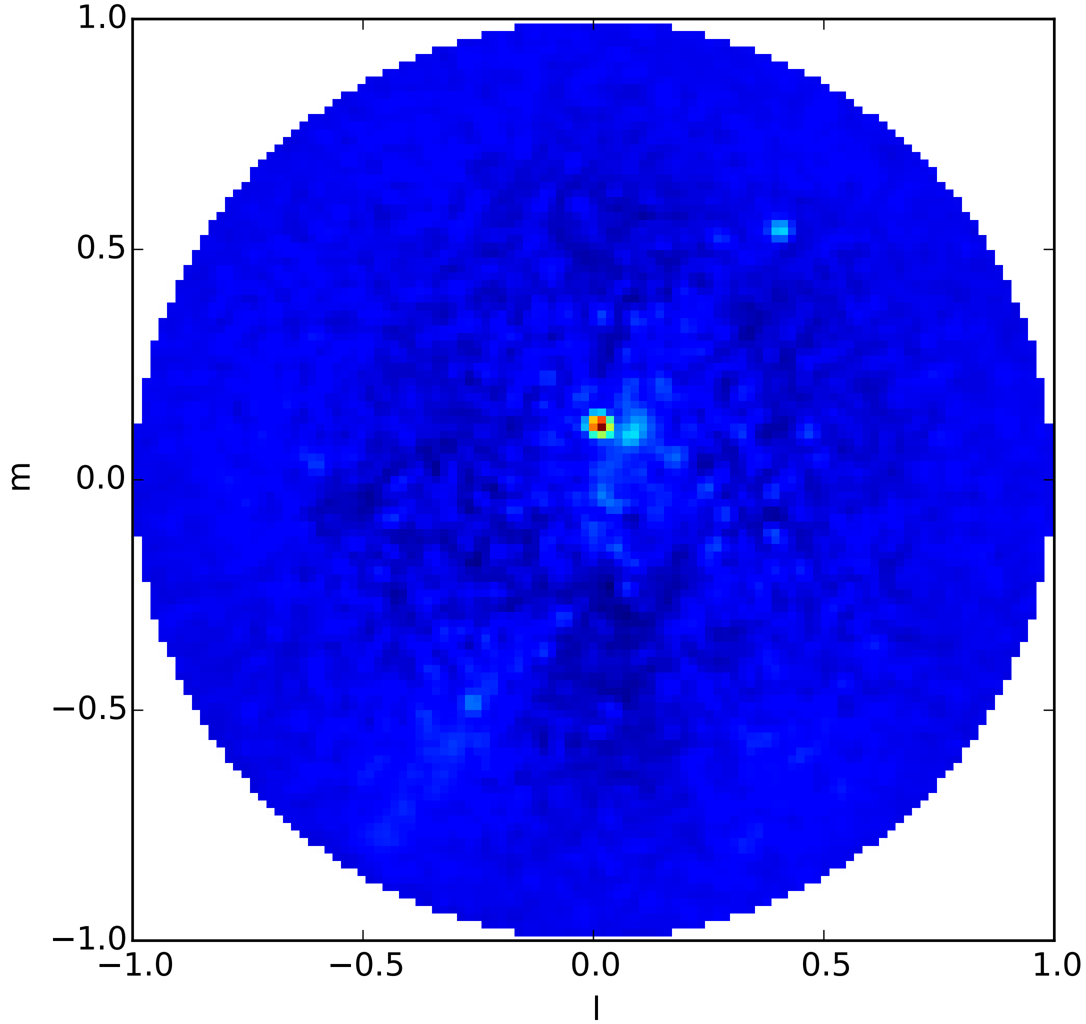
- Same data as Nithya
- I use 150 channels ~ 29 kHz bandwidth
- Cal loop on 51.2 ms cadence
    - use a total 512 ms
- Model Cyg A and Cas A as pt sources
- Need to handle auto correlation terms

Calibrating LWA data



Calibrating LWA data

Image Before Calibration



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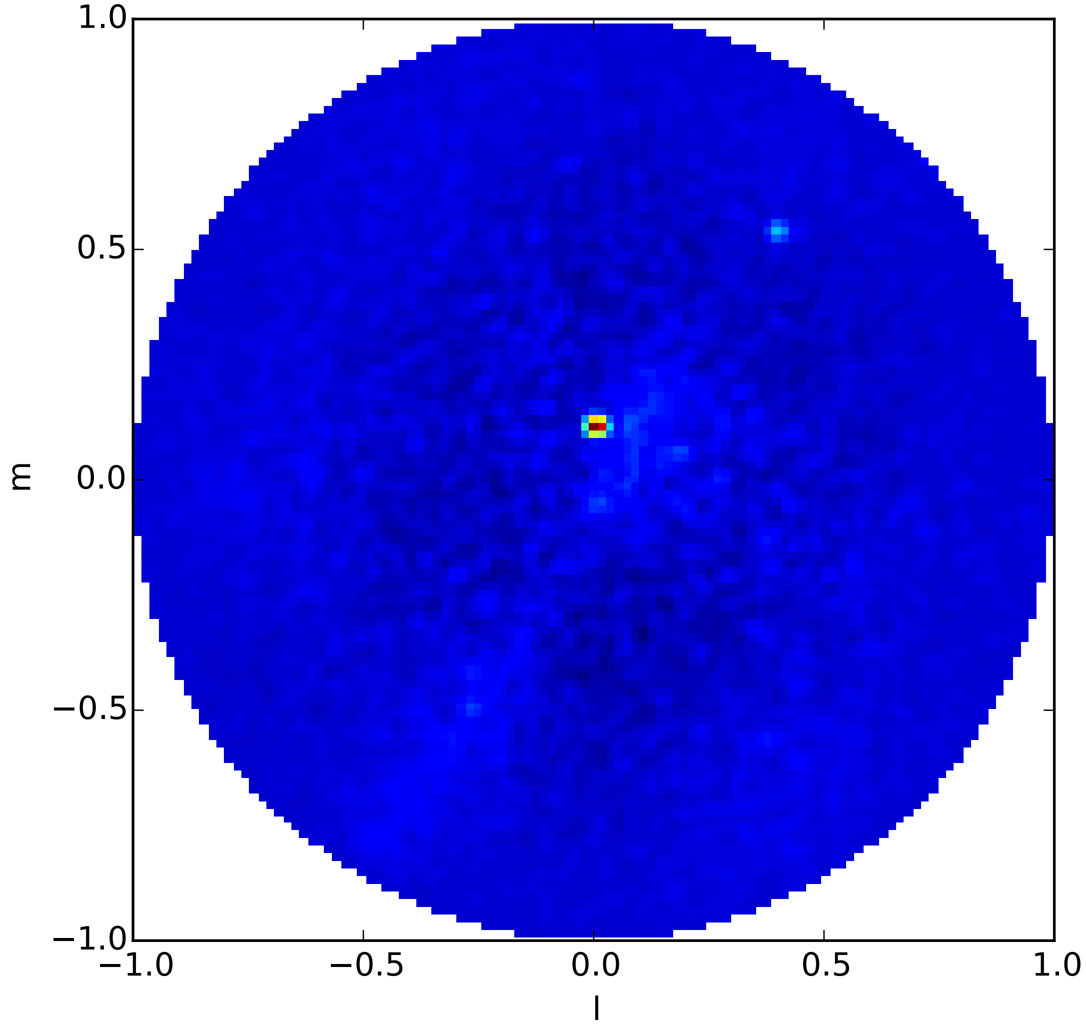
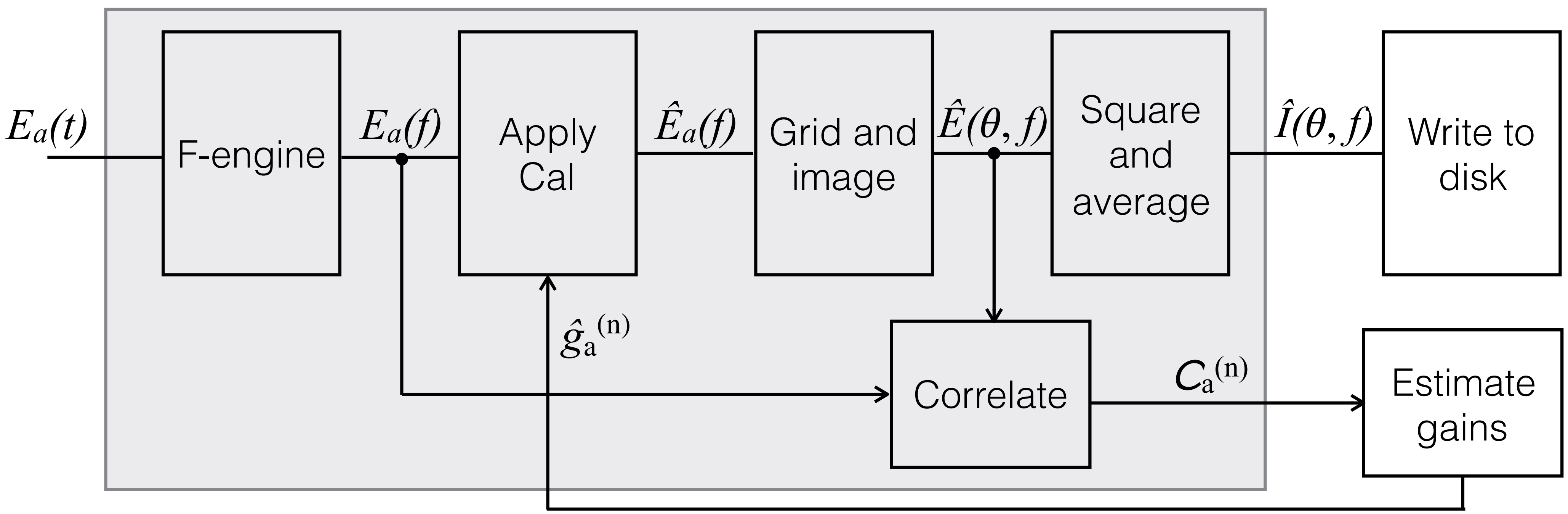


39% increase in dynamic range

Further directions



- Sophisticated fits
- Direction dependence
- Deploy on arrays
- Self-cal loop